A Comparison of Measurement Uncertainty for the Case of Non-Uniform Source Distribution between Rotating Boxes and Stationary Boxes with Multiple Detector Locations - 11182

Henrik Jäderström*
Canberra Solutions AB, Kungsgatan 107, 753 18 Uppsala, Sweden
Frazier Bronson
Canberra Industries, 800 Research Parkway, Meriden CT 06450, USA
* Corresponding author: henrik.jaderstrom@canberra.com

ABSTRACT

The computer code ISOCS (In-situ object counting system)/LabSOCS (Laboratory sourceless calibration software) has been used by hundreds of users to calculate efficiencies for mostly HPGe detectors both for in situ measurements and laboratory geometries. It is particularly useful for geometries which are difficult to calibrate with sources. Boxes filled with soil or metal scrap are examples of geometries which are difficult to calibrate using sources and often contain non homogenous source distributions. ISOCS Uncertainty Estimator (IUE) is a new tool for estimating measurement uncertainty for counting geometries which have parameters that are not well known. The IUE is based on the ISOCS code. It generates a number of models where the not well known parameters are varied in a user defined manner and calculates the efficiency for each of those models. When the average efficiency for all models converges the arithmetic and geometric standard deviations are calculated and can be used as an estimate for the measurement uncertainty for the geometry with not well known parameters. A new feature in a test version of IUE has been created to allow for boxes to be rotated around an axis that goes through the center of the box in order to simulate a box on a turntable. This new feature has been validated by comparison with good agreement to manually created models for each step of the rotation using ISOCS. Calculation time for a rotation is largely determined by the number of steps because for each step a new calculation of efficiency has to be made. The minimum number of steps necessary for accurate results has been investigated. It is of particular interest to compute the efficiency and measurement uncertainty for a measurement scenario where important parameters, e.g. the number of hotspots, are not well known for a rotating box. These results were compared to the same scenario but with one or multiple detectors in fixed position around a stationary box to try to determine the most suitable arrangement for a measuring campaign. The conclusion is that a rotating box viewed by a single detector with 20 cm clearance has about the same detection efficiency and uncertainty as two fixed detectors at 100 cm with the detectors on opposite sides of the box.

INTRODUCTION

The objective of a measuring campaign is often to determine the activity of boxes containing radioactive material. For boxes with homogenous activity concentration, well known matrix composition and box dimensions, it is straight forward to determine the efficiency with mathematical calibration software. However, if the activity is contained in small lumps or hotspots the detection efficiency depends strongly on the number of and the position of hotspots. Since the nature of this non-uniform activity is rarely known, it is normal to use a uniform homogeneous distribution of radioactivity for the calibration. But, if the object is not uniform in radioactivity, the efficiency is not likely to be correct; in such cases, the uncertainty in the efficiency calibration must be estimated and propagated with the other elements to determine the total uncertainty of the reported activity.

In a previous study of F. Bronson et.al [1], the uncertainty of boxes with various number of hotspots measured with one or more detectors positioned around the box was determined and compared to taking a sample and measuring it in the laboratory. This paper is a continuation of the above mentioned study and aims to investigate the uncertainty for boxes that are rotated around an axis going through the center of the boxes, for example putting the box on a turntable. Detector setups with one detector facing the center of the long side of the detector or two detectors positioned around the object have been simulated and the uncertainty has been determined.

The advantage of rotating the box while it is measured is that a concentration of activity can not be hidden close to the side of the box that is furthest away from the detector. One major disadvantage is that a device
for rotating the box is needed which could be costly. This study aims to give an indication whether this extra effort and cost is worthwhile.

In this study, we used the same box size and hot-spot distribution as in the previous study, therefore the sample extraction uncertainty will be the same as in the previous study [1], and was not replicated. That study concluded that the measurement uncertainty was always less than the uncertainty from any practical sampling strategy.

ISOCS UNCERTAINTY ESTIMATOR

The ISOCS [2,3,4] code calculates the efficiency for geometries that can be described by 21 different templates. Since every HPGe detector is different, crystal size, dead layer, end cap etc, the detector is characterized to achieve as good accuracy as possible. The characterization is done by source measurements and MCNP [5] calculations and gives a parameterization of the efficiency of the detector as a function of space and energy. The geometry, described by one of the templates, is divided into small volumes, voxels, and a point source is assumed to be at a random position within each of these voxels. The efficiency from every voxel is calculated from the parameterization and the attenuation from the material between the voxel and the detector. The total efficiency is the average of the efficiency from all voxels. The number of voxels is doubled and the efficiency is recalculated. The doubling and recalculation process is continued until the efficiency no longer changes significantly. The ISOCS code has been extensively validated against source measurements and it has an accuracy from 15% (10 keV) to 4% (>300 keV).

Mathematical calibrations, for example MCNP and ISOCS, are attractive to use when the measured object is large or contains materials which are not easy or accurate to simulate. However they suffer the same limitation as source calibrations: they are only valid if the measured object is the same as the one used for calibration. The ISOCS Uncertainty Estimator (IUE) [6] has been developed to estimate the uncertainty for objects that may be different from the object that is used for the efficiency calculation or if it has parameters that are not well known.

In the IUE parameters such as matrix composition, fill height, number and size and composition of hotspots, container size and wall thickness can be varied within user-defined boundaries. The IUE defines a number of models where the parameters vary independently from each other. IUE uses the ISOCS code to calculate the efficiency for all models and the efficiency is either the arithmetic or the geometric average of those models. The arithmetic and geometric standard deviation is also calculated for this group of efficiencies. The calculations are stopped when the mean efficiency or the uncertainty converges or a maximum number of models have been calculated.

BOX ROTATIONS IN IUE

This test version of the IUE has a new feature that allows for efficiency and uncertainty calculations of boxes rotating around a vertical axis in the center of the box, simulating a box on a turntable. The IUE calculates the rotation of the box by a user defined number of discrete fixed angle rotations.

The box is modeled in a geometry composer in the same way as the models are defined in ISOCS. This model is then used by the IUE as the starting point of the rotation. The rotation can be seen as moving the detector along a circle around a fixed box, illustrated in Figure 1. The box has a width w and depth t.
ISOCS uses five coordinates to describe the position of the detector relative to the object containing activity.

- Sd1, distance between the object reference plane and the detector.
- Sd2, the horizontal displacement of the detector location
- Sd3, the vertical displacement of the detector location
- Sd4, the horizontal displacement of the detector aiming point
- Sd5, the vertical displacement of the detector aiming point

Since the rotation is around a vertical axis in the center of the box the two vertical displacement coordinates are unchanged. To simulate a detector at radius r and rotation angle $\theta$ the coordinates, sd1, sd2, and sd4 are changed as follows:

$$sd1 = r \cos \theta - \frac{t}{2}$$
$$sd2 = r \sin \theta$$
$$sd4 = \frac{t}{2} \tan \theta$$

Since ISOCS models can not be defined with the detector positioned behind the object reference plane, when sd1 becomes less than zero the whole box is rotated 90 degrees and theta is decreased by the same number of degrees. The next rotation is then done from this new box. IUE then uses the ISOCS code to
calculate the efficiency of each of these models and averages the results to get the total efficiency of the rotating box.

The first step in this study was to validate that the IUE calculates the rotation correctly. This was done by comparing the efficiency from the IUE calculation for the complete rotation to average efficiency from a manual efficiency calculation at each rotation step. The ISOCS code has been extensively validated in [2] so the purpose of the comparison is only to verify that IUE rotation calculations are correct.

The box used for validating was 70 cm x 50 cm x 50 cm high stainless steel box, 2 mm wall thickness, and filled with non-radioactive dirt with one small radioactive hot spot close to one of the corners. The starting position of the detector was 100 cm from the center of the side of box.

Table 1 shows the efficiency calculated manually and by ISOCS and the difference between the two calculations for energies between 45 and 2000 keV. The difference is small, on the order of 0.1 % for high energies and below 0.01% for low energies. The main source for the difference between the calculations is different rounding of the detector positions. The difference is however well below the uncertainty of the ISOCS code which is of the order of 4-15% depending on energy.

Table 1, Efficiency calculated manually and by IUE for a rotating box

<table>
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<tr>
<th>Energy (keV)</th>
<th>Efficiency, Manual calculation</th>
<th>Efficiency, IUE calculation</th>
<th>Difference (%)</th>
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</table>

The calculation time for a rotating box in IUE depends largely on the number of steps for the rotation, since a calculation of the efficiency is done for each step. Therefore it is interesting to see how fast the efficiency converges when the number of steps is increased. In this study a box with the dimensions 1x1x2 m³ and one hotspot in one of the corners. The difference between a certain number of steps and a very large number (60) is plotted in Figure 2.
For a small number of steps the difference in efficiency varies a lot for all energies but the largest variance is for the lowest energies. It is necessary to have enough steps per rotation so that the efficiency variation is much smaller than the other uncertainties; ~20 rotations should be enough.

Uncertainty of Rotating Boxes

This study is a continuation of the study with fixed boxes [1] and is evaluating if rotating the box reduces the uncertainty of the measurement. The container is a 1m x 2m long box that is 1m tall. In this scenario the boxes to be measured contains a non-active mixture of 50% wood and 50% steel with a density of 0.5 g/cm³. The containers stainless steel walls were 3 mm thick. The activity is contained in 10x10x10 cm³ hotspots which are randomly distributed in the boxes. Five different classifications of the boxes were used, containing 1-5, 10-15, 20-30, 100 and 150 hotspots. The different number and positions of the hotspots were considered to be the main source of uncertainties and all other parameters were considered as well known.

Four different detector configurations were used in the simulations:
- One detector facing the center of the side of the box [starting position is the long side]
- Two detectors vertically; one 30 cm above and one 30 cm below the center of the box.
- Two detectors horizontally; one 60 cm to the right and the other 60 cm to the left of the center of the box.
- Two detectors opposite; each facing the center of the box but on opposite sides.

Two different distances from the box for the detectors were used – 20 cm and 100 cm. These distances were determined with respect to the rotation circle made by the corner of the box, where 0 cm is the distance where the corner of the box just clears the detector.

The uncertainty results for the rotating 1x2x1m box are shown in Table 2. Data are presented for 3 energies covering a typical measurement range. The percent standard deviation is presented for both arithmetic [%sdA] and geometric [%sdG] calculation methods.
Table 2. Uncertainty for 1x1x2 m³ rotating boxes with hotspots

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From the simulations it can be seen in Figure 3 that the uncertainty goes down with the number of hotspots; a factor of 10 increase in the number hotspots reduces the uncertainty by approximately a factor of 2.5-3, as expected.
Figure 3, Arithmetic standard deviation of 200 keV gammas as a function of number of hotspots

Figure 4 shows that as the energy increases, the standard deviation decreases. The graph only shows one geometry but the trend is typical for all three energies studied.

Figure 4, Arithmetic standard deviation for various numbers of hotspots as a function of energy

A statistical analysis of the data shows that using more than one detector does not change the uncertainty significantly. The 3 multi-detector geometries have essentially the same standard deviation as the single detector geometry at that same distance. Multiple detectors do however reduce the count time to reach a specific MDA or a specific required precision of the activity.

Increasing the distance reduces the uncertainty by approximately a factor of 1.3, but also reduces the efficiency approximately a factor of 3. The count time would have then to be increased to achieve the same MDA as for the closer distance.
Comparing with non rotating boxes [1] there are some significant differences. Most important is that the uncertainty is much lower, a factor of for 4-5 for 60 keV and 2-3 for 200 and 1000 keV. One detector at 20 cm from a rotating box has about the same uncertainty as 2-4 detectors each at 100 cm around a stationary box. See Figure 5 for the arithmetic standard deviation as a function of number of hotspots for 60 keV gammas for both rotating and fixed boxes. It can to some extent be explained by the fact that the detector is placed further away from the center of mass of the box because it needs to be able to pass the corners during a rotation. But mainly, the reduction is caused by the rotation which brings hotspots close to the detector at some point during the rotation.

![Figure 5. Arithmetic uncertainty vs energy for 60 keV gammas for rotating and fixed boxes, data are taken from [1]](image)

Having fewer detectors and having them further away from the box will of course lead to a lower efficiency and a therefore a longer counting time for a specified accuracy or MDA. To get an estimate on how big the efficiency difference is between the rotating and fixed box geometries the efficiency of a uniform box was calculated for the single detector and 2 detectors on opposite sides. The result [Figure 6] in efficiency reduction by going from a 2 fixed detectors at 100 cm to a rotating box with 1 detector at 20 cm is a factor of about 1.5; for 4 fixed detectors, the reduction in efficiency would be about a factor of 3.
Figure 6. Efficiency for one detector and 2 detectors on opposite sides for fixed and rotating boxes as a function of energy

CONCLUSIONS

From this study it can be concluded that the calculation of rotating boxes in ISOCS uncertainty estimator has been verified by manual calculation of the same rotation. Furthermore the number of steps to approach the same efficiency as a large number of steps is approximately 20 steps.

The uncertainty for rotating boxes is significantly smaller using one detector compared to a fixed box. It is comparable to using multiple detectors positioned around the measured box. Of course more detectors give a higher efficiency than using only one detector. To reduce the uncertainty one can either use multiple detectors around the box or use fewer detectors with the box on a rotating turntable.