Experimental Determination of the Multiplicity Deadtime Parameter

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ABSTRACT

Definition, extraction, and application of dead-time parameters, in correlated neutron counting, are long standing, thorny issues. Traditionally, dead-time corrections have been estimated on the assumption of a simple paralyzing model, arising from the action of the discriminator in the charged amplifiers connected to the \(^3\)He proportional counters, using a fixed dead-time. Various schemes exist to apply the paralyzable model to the multiplicity shift register histogram data. In principle, several methods could be used to estimate the dead-time parameter. The approach which is most widely applied involves measuring a series of Cf-252 sources spanning a wide dynamic range of counting rates. Ratios between the Singles, Doubles and Triples rates which ought to be independent of fissile mass are extracted. The dead-time is chosen so as to achieve the best independence in the ratios, characteristic of the fissioning system, over the counting range. These measurements can be quite laborious to conduct; require a set of Cf-252 sources matched in construction and isotopic composition; require long counts to achieve the requisite precision and involve a good deal of numerical analysis to interpret. In this work we present a simpler scheme which produces comparable values in a way that is easier to implement. In essence we place a near random neutron source, such as may be realized using Am/Li α-n sources, in the cavity and record the multiplicity histograms as one would for an assay. The variance to mean-squared is narrower than for a random counting experiment, however, as a result of the dead-time losses. A simple formula exists allowing the dead-time to be extracted from this measurement.

In this paper, we present results for the traditional approach, the proposed approach and also a variation based on adjusting the total event rate by adding Am/Li sources to a single Cf-252 source fixed in position for the case of a pair of Passive Scrap Multiplicity Counters (PSMC’s). Excellent agreement is obtained. Because the variance to mean squared approach gives high precision in a single, short, measurement it lends itself to studying dead-time behaviors systematically as a function of shift register settings, operating voltage and preamplifier trim.

INTRODUCTION

Definition, extraction, and application of dead-time parameters, in correlated neutron counting using gas filled proportional counters, are long standing problems which have not been fully resolved.
Traditionally, dead-time corrections have been estimated on the assumption of a simple paralyzing model, arising from the action of the discriminator in the charged amplifiers connected to the $^3$He proportional counters, using a fixed dead-time. Various schemes exist to apply the paralyzable model to the multiplicity shift register histogram data. In principle, a cylindrical gas filled proportional counter does not have an inherent dead-time, it simply responds to the collection of charge as it was deposited in the gas. However, when the analog signal from a charge amplifier is presented to a discriminator/logic pulse generator a processing time (at least equal to the width of the logic pulse) is created for that channel. Events that have piled-up within the charge collection time of the proportional counter can also be missed by the discriminator because the signal may not fall below the threshold to register the second (or subsequent) events. This is the origin of the paralyzability.

**DESCRIPTION OF THE NEUTRON MULTIPLICITY COUNTERS**

The counters used during this study are Passive Scrap Multiplicity Counters (Canberra Model PSMC-01). The design is based on a multiplicity counter developed by Los Alamos National Laboratory [1]. Fig. 1 shows various views of the counters. The PSMC-01 utilizes 80 $^3$He proportional tubes arranged in four concentric rings in high-density polyethylene (HDPE). The nominal assay cavity is 200 mm (7.9 inches) in diameter and 400 mm (15.7 inches) tall, lined with 1 mm (0.04 inches) of Cadmium. Graphite end-plugs are used to improve efficiency and make the axial response more uniform. The $^3$He tubes are divided into 20 individual detector banks via Amptek® JAB-01 boards. The number of tubes per bank varies by ring, in order to even out dead-time, with fewer tubes per bank in the inner most ring. The detector banks are daisy chained through the junction box via a 20 MHz, 16-bits deep de-randomizer board that serves to further reduce the counter dead-time. The main output signal is a 20 ns wide TTL pulse train from all detectors; however, output signals from each ring are also available. The TTL signal is the input for the JSR-14™ neutron multiplicity coincidence analyzer, which is accessed and controlled via the NDA 2000™ software package on a standard PC. The detection efficiency is around 54% and the die-away time is approximately 50 $\mu$s. Based on the performed optimization measurements, the PSMC units were operated with a pre-delay of 4.5 $\mu$s and a gate-width of 64 $\mu$s. The high voltage (HV) setting was 1680 V for one unit and 1660 V for the other one based on Singles, Doubles, and Triples plateau measurements. These values are consistent with the operating parameters used in previous instruments of this type.
TRADITIONAL METHOD FOR MULTIPLICITY DEAD-TIME DETERMINATION

The most wide spread approach to correcting passive neutron multiplicity counter (PNMC) data for dead-time losses is based on that described by Dytlewski [2]. The Doubles and Triples rates are derived from the observed histogram distributions using weighting factors (the so called $\alpha$ and $\beta$ arrays) based on Vincent’s loss factors, which are based on the paralysable (Type II, extendable, cumulative or updating) model. Dytlewski’s derivation, however, is not specific about the method to treat the losses in the Singles (or Trigger) rate, which determines how often the coincidence gate is opened. An approximate ad hoc expression, given in equation (1), is used where $\tau$ is the dead-time parameter.

$$S_c = e^{\tau S_n} \cdot S_m$$  \hspace{1cm} (Eq. 1)

Following the basic method of Dytlewski, the dead-time corrected Doubles and Triples neutron coincidence rates are expressed as follows in equations (2, 3, and 4).

$$D_c = \left\{ \sum_{i=1}^{n} (p_i - q_i) \cdot \alpha_i \right\} \cdot e^{\tau S_n} \cdot e^{c S_n} \cdot S_m$$  \hspace{1cm} (Eq. 2)

$$T_c = \left\{ \sum_{i=2}^{n} \beta_i (p_i - q_i) - \sum_{i=1}^{n} \alpha_i (p_i - q_i) \cdot \sum_{i=1}^{n} \alpha_i \cdot q_i \right\} \cdot e^{\tau S_n} \cdot e^{d S_n} \cdot S_m$$  \hspace{1cm} (Eq. 3)

$$\alpha_i = 1 + \sum_{j=0}^{i-2} \frac{(j-1) \phi^j}{(j+1) [1 - (j+1) \cdot \phi]^{j+2}}, \quad \beta_i = \alpha_i - 1 + \sum_{j=0}^{i-3} \frac{(i-1) \phi^j}{(j+2) [1 - (j+2) \cdot \phi]^{j+3}}.$$  \hspace{1cm} (Eq. 4)
where \( S_m \) is the non-dead-time corrected Singles rate, \( S_c, D_c, \) and \( T_c \) are the dead-time corrected Singles, Doubles, and Triples rates respectively, \( \Phi = \tau / G \), \( \tau \) is the characteristic dead-time parameter, \( G \) is the coincidence gate width, \( c \) and \( d \) are additional empirical Doubles and Triples dead-time parameters, and \( p_i \) and \( q_i \) are the normalized elements of the observed \((R+A)\) and \(A\) multiplicity histograms respectively.

Calibration for the dead-time parameters, \( c, d \) and \( \tau \), is traditionally performed by measurement of a series of \(^{252}\text{Cf}\) sources spanning the expected count rate range (typically 1 kHz to 1 MHz). Because there is no significant multiplication or \((\alpha, n)\) reaction rate with the \(^{252}\text{Cf}\) sources, the ratios of \(T/D, T/S\) and \(D/S\) should be constant, once dead-time corrected, independent of source strength. The dead-time correction parameters are then determined adjustment of the parameters to obtain the minimum chi-square value for each of the rates ratios. Typical data sets, for both counters, are provided in Table 1 and Table 2.

Table 1 Summary of Multiplicity Dead-time parameter determination data for the first PSMC-01.

<table>
<thead>
<tr>
<th>Source ID</th>
<th>Singles Rate</th>
<th>Doubles Rate</th>
<th>Triples Rate</th>
<th>D/S</th>
<th>T/S</th>
<th>T/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>G351</td>
<td>466.26</td>
<td>248.19</td>
<td>74.68</td>
<td>0.5323</td>
<td>0.1602</td>
<td>0.3009</td>
</tr>
<tr>
<td>95-4</td>
<td>940.13</td>
<td>502.34</td>
<td>151.58</td>
<td>0.5343</td>
<td>0.1612</td>
<td>0.3017</td>
</tr>
<tr>
<td>C6-256</td>
<td>15332.30</td>
<td>8198.01</td>
<td>2480.71</td>
<td>0.5347</td>
<td>0.1618</td>
<td>0.3026</td>
</tr>
<tr>
<td>Cf-003</td>
<td>18623.76</td>
<td>9792.11</td>
<td>2911.10</td>
<td>0.5258</td>
<td>0.1563</td>
<td>0.2973</td>
</tr>
<tr>
<td>Cf-01-1</td>
<td>20463.78</td>
<td>10941.36</td>
<td>3320.65</td>
<td>0.5347</td>
<td>0.1623</td>
<td>0.3035</td>
</tr>
<tr>
<td>95-4 &amp; 85-11</td>
<td>26242.90</td>
<td>517.60</td>
<td>157.73</td>
<td>0.3047</td>
<td></td>
<td></td>
</tr>
<tr>
<td>97-13</td>
<td>144138.04</td>
<td>75741.09</td>
<td>22581.47</td>
<td>0.5255</td>
<td>0.1567</td>
<td>0.2981</td>
</tr>
<tr>
<td>Cf-04-1</td>
<td>453589.82</td>
<td>241911.62</td>
<td>73311.57</td>
<td>0.5333</td>
<td>0.1616</td>
<td>0.3031</td>
</tr>
<tr>
<td>Average Ratios</td>
<td></td>
<td></td>
<td></td>
<td>0.5315</td>
<td>0.1600</td>
<td>0.3015</td>
</tr>
<tr>
<td>Standard Deviation (%)</td>
<td></td>
<td></td>
<td></td>
<td>0.77%</td>
<td>1.56%</td>
<td>0.86%</td>
</tr>
</tbody>
</table>

Parameter \( \tau \) | 46.97 ± 0.45 ns |
Parameter \( c \) | 21.8 ± 3.9 ns |
Parameter \( d \) | 21.8 ± 3.9 ns |

Table 2 Summary of Multiplicity Dead-time parameter determination data for the second PSMC-01.

<table>
<thead>
<tr>
<th>Source ID</th>
<th>Singles Rate</th>
<th>Doubles Rate</th>
<th>Triples Rate</th>
<th>D/S</th>
<th>T/S</th>
<th>T/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>G351</td>
<td>474.07</td>
<td>252.54</td>
<td>76.08</td>
<td>0.5327</td>
<td>0.1605</td>
<td>0.3013</td>
</tr>
<tr>
<td>95-4</td>
<td>953.09</td>
<td>508.22</td>
<td>152.82</td>
<td>0.5332</td>
<td>0.1603</td>
<td>0.3007</td>
</tr>
<tr>
<td>C6-256</td>
<td>15539.20</td>
<td>8307.75</td>
<td>2518.97</td>
<td>0.5346</td>
<td>0.1621</td>
<td>0.3032</td>
</tr>
<tr>
<td>Cf-003</td>
<td>18851.54</td>
<td>9897.81</td>
<td>2919.42</td>
<td>0.5250</td>
<td>0.1549</td>
<td>0.2950</td>
</tr>
<tr>
<td>Cf-01-1</td>
<td>20772.03</td>
<td>11096.11</td>
<td>3354.65</td>
<td>0.5342</td>
<td>0.1615</td>
<td>0.3023</td>
</tr>
<tr>
<td>95-4 &amp; 85-11</td>
<td>25803.10</td>
<td>516.16</td>
<td>156.60</td>
<td>0.3034</td>
<td></td>
<td></td>
</tr>
<tr>
<td>97-13</td>
<td>145864.40</td>
<td>76619.17</td>
<td>22767.86</td>
<td>0.5253</td>
<td>0.1561</td>
<td>0.2972</td>
</tr>
<tr>
<td>Cf-04-1</td>
<td>460355.69</td>
<td>245323.99</td>
<td>74460.34</td>
<td>0.5329</td>
<td>0.1617</td>
<td>0.3035</td>
</tr>
<tr>
<td>Average Ratios</td>
<td></td>
<td></td>
<td></td>
<td>0.5311</td>
<td>0.1596</td>
<td>0.3008</td>
</tr>
<tr>
<td>Standard Deviation (%)</td>
<td></td>
<td></td>
<td></td>
<td>0.78%</td>
<td>0.78%</td>
<td>1.82%</td>
</tr>
</tbody>
</table>

Parameter \( \tau \) | 46.22 ± 0.45 ns |
Parameter \( c \) | 22.0 ± 3.9 ns |
Parameter \( d \) | 22.0 ± 3.9 ns |
Historically, we have found that setting the parameter c = d, during the optimization, usually yields the lower reduced $\chi^2$ value but otherwise there is no definitive reason to select this approach for correction of the Doubles and Triples rates.

Other methods of multiplicity dead-time determinations are given in Vincent [3] and Hage and Cifarelli [4]. A thorough overview of the neutron coincidence and multiplicity dead time determination along with a suggested method for multiplicity determination is given in McElroy et. al. [5] and Croft et. al. [6]

Vincent [3] considered the problem of making multiplicity dead-time corrections within the paralyzable model taking into account time correlations but his results do not appear to have been adopted by the community or even independently applied and evaluated which appears to be an omission.

Also, Hage and Cifarelli [4] developed a detailed mathematical model as to how to correct multiplicity data within the framework of a paralyzable neutron dead-time system with simple exponential die-away profile. This approach also appears to be worthy of more widespread use subject to more taxing evaluation at higher rates than has hitherto been the case and with state of the current practice detectors. The dead-time corrections for all moments depend on the multiplicity distribution. The treatment, although elegant and masterful, is also compact and technical which may partially explain why it has not percolated more widely. In common with all treatments however, the approach considers an idealized detector head. The dead-time parameters for a system are typically determined experimentally by loading the detector chains approximately symmetrically. In application deviation form, this pattern will emphasize on preamplifier/processing circuit more than another and lead to slightly different losses. Possibilities, such as this, suggest more complete data acquisition information may prove useful in future endeavors to understand and more completely simulate rate loss effects.

The next section provides a new method for dead time determination in this avenue and is based on the statistical properties summarized by Para and Bettoni [7].

**PROPOSED METHOD**

The proposed dead-time correction method is based on dead-time affected statistical distributions as set out by Para and Bettoni [7]. Detectors, in nuclear counting, can be categorized into two main classes: paralyzable and non-paralyzable. The classification of a counter depends on the dead-time losses behavior. In the non-paralyzable case, every detected pulse is followed by a certain dead-time period $\tau$, in which every incoming pulse is lost. For paralyzable counters, every pulse falling during the dead-time period following a detected pulse is not lost but rather extends the dead-time period by the same time period $\tau$.

Neutron counting can be classified further based on the status of the counter when receiving the first pulse. The “free” case where the counter started counting while it was not in a dead-time loss state due to a previous pulse just prior to the start of the counting. The “not-free” case occurs when the counter starts counting while it was inhibited from counting due to a previous pulse. In reality, the true dead-
time loss behavior falls between the two classes. None-the-less, to a good approximation, dead-time losses of neutron counters can be derived using the extreme case where we have paralyzability and “not-free” counter.

According to Para and Bettoni [7], the probability of registering “k” events in the (0, t) time interval, in the paralyzable and “not-free” counter case, is given by equation (4), where $\tau$ is the dead-time parameter associated with each count and “m” is the true time- averaged expectation value for the number of countable events; the mean number of counts in the absence of dead-time losses.

$$P\{n(0,t) = k\} = \sum_{j=k}^{\infty} (-1)^{j-k} \frac{e^{mj} e^{-mt}}{(j-k)!} m^{j} (t - j \cdot \tau)^{j}$$  \hspace{1cm} (Eq. 4)

From equation (4), the mean value $<k>$ and the variance $\sigma^2$ can be derived using Laplace transforms. We find:

$$<k> = m \cdot \tau \cdot e^{-m \cdot \tau}$$ \hspace{1cm} (Eq. 5)

$$\sigma^2 = m \cdot \tau \cdot e^{-m \cdot \tau} + m^2 \cdot (\tau^2 - 2 \cdot t \cdot \tau) \cdot e^{-2 \cdot m \cdot \tau}$$  \hspace{1cm} (Eq. 6)

In principle, one may fit the theoretical number distribution, given by equation (4), to the experimental data as a means of extracting the dead-time parameter $\tau$. Solving equations (5) and (6) for $\tau$ and defining $\phi = \tau / t$, we get:

$$\phi = 1 - \sqrt{1 - \left[ \frac{<k> - \sigma^2}{<k>^2} \right]}; \phi = \frac{\tau}{t}$$ \hspace{1cm} (Eq. 7)

$<k>$ and $\sigma^2$ are the mean value and the variance of the measured number distribution respectively. They are directly measured from the Accidentals multiplicity distribution (which for a random source is statistically equivalent to the (R+A) distribution provided that the pre-delay parameter is suitably set) and defined in equations [8] and [9] for a given mathematical distribution $A_i$:

$$<k> = \frac{\sum_i i \cdot A_i}{\sum_i A_i}$$ \hspace{1cm} (Eq. 8)

$$\sigma^2 = \frac{\sum_i [i - <k>]^2 \cdot A_i}{\sum_i A_i}$$ \hspace{1cm} (Eq. 9)

Measurements were conducted through NDA 2000™ using a gate width of 64 us and 10 Am-Li sources with combined source strength of about 10 Ci corresponding to an apparent trigger rate of approximately $3 \times 10^5$ n/s. The Am/Li sources were placed around the center of the PSMC-01 counters. Using NDA 2000™, we acquired eight 1-hour count-time multiplicity distributions using the Canberra JSR-14 analyzer. Both the mean and the variance of each Accidentals multiplicity distribution were extracted and used to solve for the multiplicity dead-time parameter. The extracted dead-times for each
1-hour Accidentals multiplicity distribution for one of the PSMC-01 units are shown in Fig. 2. The same measurements were performed on the second PSMC-01 counter. A comparison of the new multiplicity dead-times and those obtained through the standard method given in the previous section is presented in Error! Reference source not found. Besides the excellent agreement between the traditional and proposed dead-time determination methods, the new technique showed stability for the eight 1-hour runs as clearly seen in Fig. 2. It is worth mentioning also that the new technique is able to determine the multiplicity dead-time in a fairly short amount of counting time as compared to the traditional method. This makes a systematic study of dead-time behaviors in a reasonable counting time viable.

<table>
<thead>
<tr>
<th>Counter</th>
<th>Traditional Method dead-time [ns]</th>
<th>Proposed Method dead-time [ns]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSMC01-1</td>
<td>46.9 ± 0.5</td>
<td>46.2 ± 0.3</td>
</tr>
<tr>
<td>PSMC01-2</td>
<td>46.2 ± 0.5</td>
<td>46.7 ± 0.3</td>
</tr>
</tbody>
</table>

Fig. 2. Multiplicity dead-time determination using the Accidentals distribution for 8 1-hour count-time for the first PSMC-01 counter. Measurements were conducted through NDA 2000™ using a gate width of 64 us and 10 Am-Li sources with combined source strength of about 10 Ci. The filled-diamond data points represent the extracted dead-time and the continuous line is a linear fit to the data.
The effect of varying the gate width was also studied on both counters with the pre-delay and HV set as previously mentioned. The gate width study results are summarized in Table 4.

Table 4 Multiplicity dead-times for different gate widths using the proposed method. The pre-delay was set to 4.5 us and the HV was set to the optimum value previously determined (i.e. 1680 V for the first PSMC01 and 1660 V for the second PSMC01). The reported uncertainties on the dead-time parameters are only statistical.

<table>
<thead>
<tr>
<th>Gate Width [us]</th>
<th>First PSMC01</th>
<th>Second PSMC01</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>-</td>
<td>48.7 ± 0.2</td>
</tr>
<tr>
<td>16</td>
<td>47.6 ± 0.1</td>
<td>47.5 ± 0.1</td>
</tr>
<tr>
<td>32</td>
<td>47.1 ± 0.3</td>
<td>46.8 ± 0.3</td>
</tr>
<tr>
<td>64</td>
<td>46.2 ± 0.3</td>
<td>46.7 ± 0.3</td>
</tr>
<tr>
<td>128</td>
<td>46.5 ± 0.6</td>
<td>46.5 ± 0.5</td>
</tr>
<tr>
<td>256</td>
<td>-</td>
<td>46.2 ± 0.6</td>
</tr>
</tbody>
</table>

We conclude from the data shown in Table 4 that there is a slight dependence of the dead-time for different gate widths. This leads us to think that there are interesting relationships between the dead-time and the basic performance parameters of the neutron counters. This effect has been also been observed in ref. [6].

The dependence of the dead-time parameter on the HV setting was also studied because of the simplicity of the needed measurements for the proposed dead-time determination method. Using the combined 10 Am-Li sources and the optimum setting parameters for pre-delay and gate width, we extracted the dead-time parameter for different high voltages. One would expect the dead-time to be higher for lower operating high voltages since the neutron events have to produce signals that cross the discriminator threshold in order to be considered as a pulse as lower high voltages tend to produce smaller amplitude signals. In a sense, the lower high voltage inhibits the distribution from “growing” freely; an effect that manifests itself as a dead-time in our case. The proposed method cannot be applied for high biases that introduce gamma breakthroughs. In this case, the generated events are correlated making our model not applicable (but this would not be a suitable operating state for practical measurements). Fig. 3 shows the behavior of the dead-time, calculated with the proposed method, as the operating high voltage varies.
Fig. 3. Observed dependence of multiplicity dead-time parameter using the proposed method for different high voltages. All the other setting parameters were set to the optimal values. This data is from the first PSMC01 counter. The arrow indicates the optimum operating high voltage. (i.e. 1680V)

This data shows some rather interesting relationships between the dead-time and the basic performance parameters of the neutron counters. These behaviors suggest that the dead-time correction models used for multiplicity analysis may not be entirely robust but rather are specific to the application.

Another aspect that was studied is the effect of varying the Am-Li source strength on the dead-time determination. The data is shown in Fig. 4. We would like to warn the reader though that the proposed method relies on measuring both the mean and variance of the Accidentals distribution for dead-time determination. Due to the finite time resolution of the JSR-14 shift register (i.e. 4 MHz clock), the multiplicity distributions tend to be under-described for low rates as compared for high rates. It is suggested to measure those values for high enough trigger rate. Obviously, for faster shifter registers, such as the Canberra HHMR/JSR-15 (Hand Held Multiplicity Register), the multiplicity distributions are very well shaped even at low trigger rates [8].
In conclusion to this section, it is worth mentioning that the proposed method is able to determine the multiplicity dead-time parameter from a single short assay measurement using only Am-Li sources. The assay time is of the order of 1800 seconds corresponding to the activity of the used sources.

**PROPOSED METHOD FOR DOUBLES AND TRIPLES MULTIPLICITY DEAD-TIME CORRECTIONS**

In the previous section, the multiplicity dead-time parameter $\tau$ was extracted using only Am-Li sources. In principle, the Singles can be corrected for dead-time effect, to first order, using equation (10). The “c” and “m” indices refer to dead-time corrected and uncorrected values respectively. Consequently, the Singles dead-time correction factor is reasonably well established.

$$S_c = S_m \cdot e^{\delta_S} , \quad CF_c = e^{\delta_S} \quad \text{(Eq. 10)}$$

In order to establish the Doubles and Triples dead-time correction factors, a series of measurements based on adjusting the event rate by adding Am/Li sources to a single Cf-252 source were performed. The Cf-252 source was kept fixed in the center of the first PSMC01 counter during the assay measurements. The counter was operated at 1680 V with pre-delay and gate width of 4.5 us and 64 us respectively. 8 identical Am-Li sources of activity around 1Ci were added one at a time to a single Cf-252 with a neutron emission rate of 1.1341E+6 n/s at the time of the measurements. It is referred to as C5-134 in the present text. For each configuration the multiplicity distribution was recorded for an assay time of 1800 seconds.

![Fig. 4. Dead-time parameter determination for various Am-Li sources for trigger rate variation. The data is acquired using the first PSMC01 and optimum operating parameters settings. The uncertainties on the data points are statistical only.](image)
First, the dead-time uncorrected Singles, Doubles, and Triples are calculated from both the acquired Reals-plus-Accidentals (R+A) and Accidentals (A) distributions using equations (11), (12), and (13).

\[
S_m = \sum_{i=0}^{\text{max}} A_i / t_{\text{assay}} \quad \text{(Eq. 11)}
\]

\[
D_m = \sum_{i=1}^{\text{max}} i \cdot [(R + A)_i - A_i] / t_{\text{assay}} \quad \text{(Eq. 12)}
\]

\[
T_m = \sum_{i=1}^{\text{max}} \frac{i \cdot (i - 1)}{2} [(R + A)_i - A_i] / t_{\text{assay}} - S_m \cdot D_m \cdot T_{\text{gate}} \quad \text{(Eq. 13)}
\]

The Doubles can be dead-time corrected for using equation (14) and vary the Doubles dead-time parameter “n” such that the Doubles stays constant for various event rates. This is possible since the Am-Li sources are quasi-random sources adding negligible amount of Doubles to the number of Doubles mainly originating from the Cf-252 source.

\[
D_c = D_m \cdot e^{\delta S_c} \cdot e^{n \delta S_c} ; \quad CF_D = e^{\delta S_c} \cdot e^{n \delta S_c} \quad \text{(Eq. 14)}
\]

Fig. 5 shows the uncorrected Doubles and the effect of varying the Doubles dead-time parameter to get a constant Doubles rate. The optimization gives \( n = 2.96 \). This result is in very good agreement with the neutron coincidence counting (NCC) expectation where the Doubles correction factor is close to being the Singles correction factor raised to the fourth power. This behavior is not imposed by the model nor is it forced during the proposed analysis.

![Fig. 5](image-url)
At this point of the proposed technique we established both the Singles and Doubles corrections factors to an acceptable extent. The correction of the Triples is more complicated. The dead-time uncorrected triples can be written in the form given in equation (15). The corrected Triples can be written as shown in equation (16). The main tasks in what follows can be summarized in the determination of the correction factor CF_H as a function of the Singles rate.

\[
T_m = S_m \cdot H_m - S_m \cdot D_m \cdot T_{\text{gate}} \quad \text{(Eq. 15)}
\]

\[
H_m = \frac{\sum_{i=1}^{\max} \frac{(i-1)}{2} [(R + A)_i - A_i]}{\sum_{i=0}^{\max} A_i}
\]

\[
T_c = S_e \cdot CF_H \cdot H_m - S_e \cdot D_e \cdot T_{\text{gate}} \quad \text{(Eq. 16)}
\]

Equation (16) can be written under the form given in equation (17). The only unknown value in the equation is \(T_C\) (i.e. the dead-time corrected Triples). Due to the same reason explained above, the Triples rate is expected to be constant across the measurements. It is estimated by performing an extrapolation of the data shown in Fig. 6. The intercept with low or zero Singles establishes the dead-time corrected Triples to a good approximation. Obviously, this introduces a systematic uncertainty to the proposed technique.

\[
CF_H = \frac{T_c + S_e \cdot D_e \cdot T_{\text{gate}}}{S_e \cdot H_m} \quad \text{(Eq. 17)}
\]

Fig. 6. The above plot shows the dead-time uncorrected Triples as a function of the dead-time uncorrected Singles. The filled diamonds represent the dead-time uncorrected Triples. The intercept of the linear extrapolation establishes the dead-time corrected Triples. The data was acquired with the first PSMC01 using the optimum operating parameter settings.

The dead-time corrected Triples rate is estimated to be equal to 78012 Hz. This value can be injected into equation (17) to extract the behavior of the correction factor \(CF_H\) as a function of the dead-time.
corrected Singles as shown in Fig. 7. Its functional form is expected to be in the form given in equation (18).

\[
CF_H = (1 + m \cdot \delta \cdot S_c)
\]  
(Eq. 18)

![Graph](image)

**Fig. 7.** The above plot shows both the Triples dead-time correction factor as a function of the dead-time corrected Singles. The filled squares represent the data and the continuous line is a linear fit to the data points. The data was acquired with the first PSMC01 using the optimum operating parameter settings.

Fitting the data given in Fig. 7 determine the Triples dead-time parameter “m”. The linear fit provides \( m = 4.53 \). The behavior of \( CF_H \) is very linear with an intercept fairly close to unity.

**CONCLUSIONS**

Dead-time correction plays a vital role in any multiplicity counting analysis. We presented both the results of the traditional and proposed methods for dead-time correction. Both methods showed excellent agreement with each other.

The proposed method relies on the fact that the variance to mean-squared is narrower than for a random Am-Li source counting experiment as a result of the dead-time losses. A simple formula
exists allowing the dead-time to be extracted from this measurement. The Doubles and Triples dead-
time correction is based on adjusting the total event rate by adding Am-Li sources to a single Cf-252
source fixed in position.

The proposed new dead-time method for multiplicity counting is simple and does not require long
assay measurement times. The method corrects sequentially for the Singles, Doubles, and then the
Triples. It was applied on a pair of Passive ScrapMultiplicity Counters (PSMC01).

We hope this effort can help generate more interest and exchange of ideas on the important topic of
dead-time corrections.

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