THE EFFECT OF COUNT TIME ON RELIABLY DETECTABLE ACTIVITY (RDA) WHEN ASSESSING CONTAMINATION MONITOR PERFORMANCE

N. L. Hassig, Environmental Strategies
J. W. Wanless and L. J. Phillips, National Nuclear Corporation

ABSTRACT

Radiation monitoring equipment can "see" or detect only source activity greater than background. This paper explores the methods used to determine the minimum detectable activity (MDA) of such equipment. Under experimental conditions, known sources of low-level radiation were counted. The predetermined MDA was compared to the lowest level of known sources detectable by the equipment. It is shown that most currently used MDA equations correctly predict the minimum detectable source level. However, the MDA levels predicted by theoretical equations, without taking excess variability (high background, unstable background) into consideration, may be lower (i.e., indicate smaller sources are detectable) than is actually possible with the equipment and measuring procedures as defined.

INTRODUCTION

Measuring low level or background-dominant radioactivity requires a description of the radiation counting device, a defined measurement procedure and a statistical model for interpreting the count data. All of these factors contribute to the definition of the detection limit of the measurement process. With increased interest in monitoring of nuclear plant discharges, especially low-level radioactive wastes, valid detection limits are very important.

The chemical and radioactivity measurement literature contains a plethora of mathematical expressions and widely-ranging terminology. Minimum detectable activity (MDA), sensitivity, lower limits of detection (LLD), and signal-to-noise ratio are some of the terms currently in use. In his definitive work in this area, L. A. Currie (1) compared some of the more commonly-used definitions, and calculated "detection limits" for a hypothetical radioactivity experiment. A long-lived gamma emitter was counted for ten minutes with an efficiency of 10%, using a detector having a background of 20 cpm. The results, plotted in increasing order, encompassed nearly three orders of magnitude.

This paper reports on an experiment to empirically determine an MDA. It also answers questions such as: Are the MDA equations different? Which one is "correct"? Can smaller and smaller sources be detected just by increasing the count time?

Starting with a known low-level activity source, larger and larger sources were counted with a volumetric radiation monitor until the criteria specified by the MDA definitions were met. Sensitivity analysis was performed using various levels of background and total (background plus source) count times, background levels (low vs. high), and variability assumptions (non-Poisson vs. Poisson). The result was validation of the "commonly" used MDA equations and an understanding of the importance background levels and variability play in the limits of radiation detection.

STATISTICAL VARIABILITY

The process of radioactive decay is a random sequence in time. The radioactive decay law gives the disintegration rate dN/dt to be

$$-\frac{dN}{dt} = +\lambda N \qquad \text{(Eq. 1)}$$

where λ is the decay constant and N is the number of unstable nuclei. If n is the number of atoms which disintegrate during a certain period of time T, where N is very large such that the disintegration rate can be considered to remain constant, say p, then the probability of exactly k disintegrations during this period T can be approximated by the Poisson distribution:

$$\text{Probability}(n=k) \leq (pT)^k \frac{e^{-pt}}{k!} \qquad \text{(Eq. 2)}$$

The Poisson distribution is also applicable to a large number of physical phenomena, such as telephone calls, traffic accidents, flaws in a manufacturing process, etc.

Once we accept that the phenomenon of radioactivity has a Poisson distribution, several useful results follow by deduction:

a. The mean and the variance of the number of disintegrations during time T are both equal to pT. The standard deviation is $\sqrt{pT}$.

b. If n_1 is the number of disintegrations of one species of nuclide in time T with disintegration rate p_1 and n_2 is the number of disintegrations of another species of nuclide in time T with disintegration rate p_2, then the distribution of the sum $(n_1 + n_2)$ is Poisson with mean and variance $(p_1 + p_2)T$.

c. For a large number of disintegrations (at least 70), the Poisson distribution is well-approximated by the

Gaussian, or normal, distribution with mean pT and variance pT.

These three results are used in the definition of the limits of detection.

In addition to this predictable Poisson variation, there is also excess (non-Poisson) variability. Some of this excess variation may be systematic: instrument drift, geometry correction, spurious counts, coincidence losses, errors in counter efficiency calculations, etc. This systematic error can be so large as to completely mask the Poisson statistical variability, thus making hypothesis testing unreliable. The statistical framework of "hypothesis testing" is the theoretical basis for the MDA calculations (2).

Non-Poisson variability can be detected by making repetitive measurements over a constant time interval and comparing the sample standard deviation to that which would be expected under the Poisson model., A statistical test (chi-square) will quickly indicate excess non-Poisson error. If such excess error is present during the counting process, Currie (3) suggests using the sample standard deviation instead of the Poisson value $\sqrt{mean\ counts}$.

Excess non-Poisson variability was found during the empirical MDA testing reported on in this paper. Results from an experiment designed to determine an empirical MDA are reported in two ways: 1) using the Poisson value in the MDA calculations and 2) using the sample standard deviation. The differences are dramatic. The MDA increased by a factor of two to five (depending on count time and source strength) when the sample standard deviation was used instead of the Poisson value.

MDA DEFINITIONS AND EQUATIONS

Following the lead of Currie, most radiation counting analysts now report three levels for the lower limits of a measurement process:

1. a decision limit (called the "critical level" or L_c) which must be exceeded before the sample of the source in question can be said to contain any radioactive material above background,
2. a detection limit (L_d) at which we can be reasonably sure (usually 95%) that material having this amount of radioactivity will be detected, and
3. a determination limit (L_q) at which we can quantitatively identify the level of radioactivity in the material with a given precision. Fig. 1 graphically displays these three levels. As can be seen, L_c L_d L_q.

Using the MDA calculations developed by Currie (1) and Lochamy (4), we see that very simple derivations are presented (Fig. 2). These simple derivations belie more complicated derivations by making certain assumptions and by combining multiplicative factors. For the purposes of this empirical MDA search, these formulations were used with the slight modification of using 10% false positive (alpha) and 10% false negative (beta) instead of the 5% levels conventionally used. This modification was used to simplify the repetitive measurement process.

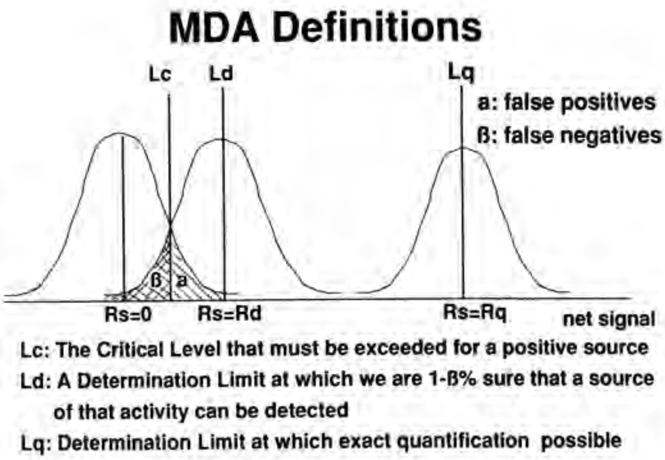

Fig. 1. Graphical Display of MDA Equations and Definitions.

MDA Equations

Critical Level (Lc): a posteriori, f (a)

Lc (counts) = 2.33 * SQRT (B/G counts)

Lc(CPS) = 2.33 * Stnd Dev (B/G rate)

Detection Limit (Ld): a priori, f (Lc, a, ß)

Ld (counts) = 2.71 + 4.66 * SQRT (B/G counts)

Ld (CPS) = (2.71/T) + 4.66 * SQRT (B/G rate)

Detection "Decision"

If Rs > Lc, sample is reported as positive activity with a two-sided confidence interval

If Rs ≤ Lc, a "less than" activity value is reported with a one-sided confidence interval

C.I. = Rs ± k * Stnd Dev (Rs)

Fig. 2. MDA Equations and Decision Rules in Common Use.

REPORTING THE DETECTION LIMITS

In practice, background (B) and background plus source (G) are counted for a predetermined period of time and a net signal ($R_S = R_G + R_B$) is calculated along with its standard deviation

$$\sigma_S = \sqrt{\sigma_G^2 + \sigma_B^2} \quad \text{(Eq. 3)}$$

L_c, L_d and L_q are calculated based on the background level and standard deviation. Two conditions are possible: 1) If $R_S > L_c$, the sample is reported as having radioactive material above background. A two-sided confidence interval is reported,

$$R_S \pm k_1\sigma_S \quad \text{(Eq. 4)}$$

where $k_1 = 1.96$ is based on the normal assumption and a 95% confidence level. 2) If $R_S <= L_c$, the sample is reported as having "less than" an upper limit value (one-sided confidence interval),

$$R_S + k_2\sigma_S \quad \text{(Eq. 5)}$$

where $k_2 = 1.65$ and is also based on the normal assumption and a 95% confidence level.

Lochamy reports that the detection limit (L_d) and the determination limit (L_q) are not used for routine counting and reporting. In those cases where one is required to specify a minimum detectable activity (e.g., to a regulatory agency), it is "recommended that the Detection Limit (L_d) be given then as a practical reporting limit".

The determination limit (L_q) is useful when a "sensitivity" with a specified relative standard deviation is required.

One note of caution: Detection limits can be specified in terms of raw counts or in terms of a rate (e.g., counts per second or cps). The Poisson assumption that the standard deviation is equal to the square root of the mean is valid only when dealing with counts

$$\sigma_{Counts} = \sqrt{\text{counts}} \quad \text{(Eq. 6)}$$

The standard deviation of a rate is

$$\sigma_{Rate} = \frac{\sigma_{Counts}}{T} = \frac{\sqrt{\text{counts}}}{T} = \sqrt{\frac{\text{rate}}{T}} \quad \text{(Eq. 7)}$$

This puts the counting interval (T) in the denominator, thus affecting the MDA calculation.

We can now answer one of the questions posed by this research project: Does increasing the counting time interval decrease the MDA? Yes at least according to theory. L_c (in counts) is a function of the standard deviation of the background counts, thus is proportional to the square root of the background counts which are themselves proportional to the counting interval. L_c (in counts per second) is a function of the standard deviation of the background count rate and thus is proportional to $1/\sqrt{T}$ or one over the square root of the counting interval, assuming the rate is constant. As the MDA varies with the square root of time (counts) or indirectly with time (rate), increasing the counting interval results in a decreased MDA.

This is what theory tells us. Unfortunately, increasing the counting interval also increases the likelihood of experiencing a change in background. Changing background level and/or variability introduces excess variability not accounted for in the statistical models on which the MDA calculations are based. To test for excess non-Poisson background variability, the chi-square test, referred to earlier, can be used. The background is very susceptible to change under most counting conditions, such as in a waste monitoring area near other nuclear plant operations. To test for Poisson background variability, the background must be tested before and after a sample count is taken. The assumption that background is stable, when it is not, and using the MDA equations without modification, will lead to a lower (incorrect) MDA.

SEARCH FOR AN EMPIRICAL MDA

To test the validity of the MDA calculations from theory, we conducted an experiment. Known, low-activity sources of Co-60 were counted in a volumetric contamination-monitoring device made by National Nuclear Corporation, called a Waste Curie Monitor. The equipment was calibrated for Co-60 and had a 39% counting efficiency. Counting was done in a low-background environment where the background was stable, probably more stable than actual plant monitoring conditions.

Based on the assumption of questionable background variability, the background was tested repeatedly between source counting. MDAs were calculated using the Poisson assumption (stable background variability) and the non-Poisson assumption (actual sample variability used in place of Poisson variability). Sample results (R^S) were calculated and compared to the critical level (L_c). Starting with small .8 nCi Co-60 sources, increasing levels of activity sources were counted up to a limit of 3.2 nCi of Co-60. False positives occur when comparison indicates a positive source but the counter is empty and only background is present.

TABLE I
MDA Calculations: False Positives Increasing Count Time.

Count Time (Sec)	Source Strength	Critical Level Lc (CPS)	# Times Rs > Lc (false positives)
10	0	19	0 / 10
30	0	10	2 / 10
60	0	8	7 / 10
180	0	5	2 / 10
210	0	4	3 / 10

False negatives occur when comparison indicates no activity above background but a source is in the counter.

Table I shows false positive results. Count times of 10 to 210 seconds were tested. Note: 210 seconds was the minimal count time required to give precise (10% relative error) results; see Loevinger and Berman (5) or Donn and Wolke (6) for a discussion of optimal count time. It can be seen that Lc decreases as count time increases. From our theoretical MDA calculations, we expected 10% false positives. We found 30% false positives. The conclusion is that we have excess background variation and the "true" MDA is larger than we would expect from the theoretical calculations. This suggests using the sample standard deviation rather than the Poisson standard deviation to calculate L_c because of the excess non-Poisson variability.

Table II shows false negative results. It shows the sensitivity of the MDA calculations to source strength and count time. We counted larger and larger activity sources for longer and longer count times. We finally achieved 0% false negatives at a source strength of 2.0 nCi counted for 210 seconds. Our theoretical MDA calculations used 10% error levels. Our conclusion is that we can assume the "true" MDA is between 1.2 nCi (30% false negatives) and 2.0 nCi (0% false negatives).

TABLE II
MDA Calculations: False Negatives

Source Strength	Count Time (sec)	# Times Rs < Lc (false Negatives)
.8	10	2 / 10
2.0	10	2 / 10
2.8	10	0 / 10
.8	30	5 / 10
.8	90	4 / 10
.8	210	4 / 10
1.2	210	3 / 10
2.0	210	0 / 10
3.2	210	0 / 10

To answer the question of whether the theoretical MDA equations are "correct", we show some of the confidence intervals and "less than" values as calculated from the results of the comparison between R_S and L_c, using the known sources and increased count times.

In Table III, we see that all of the calculated confidence

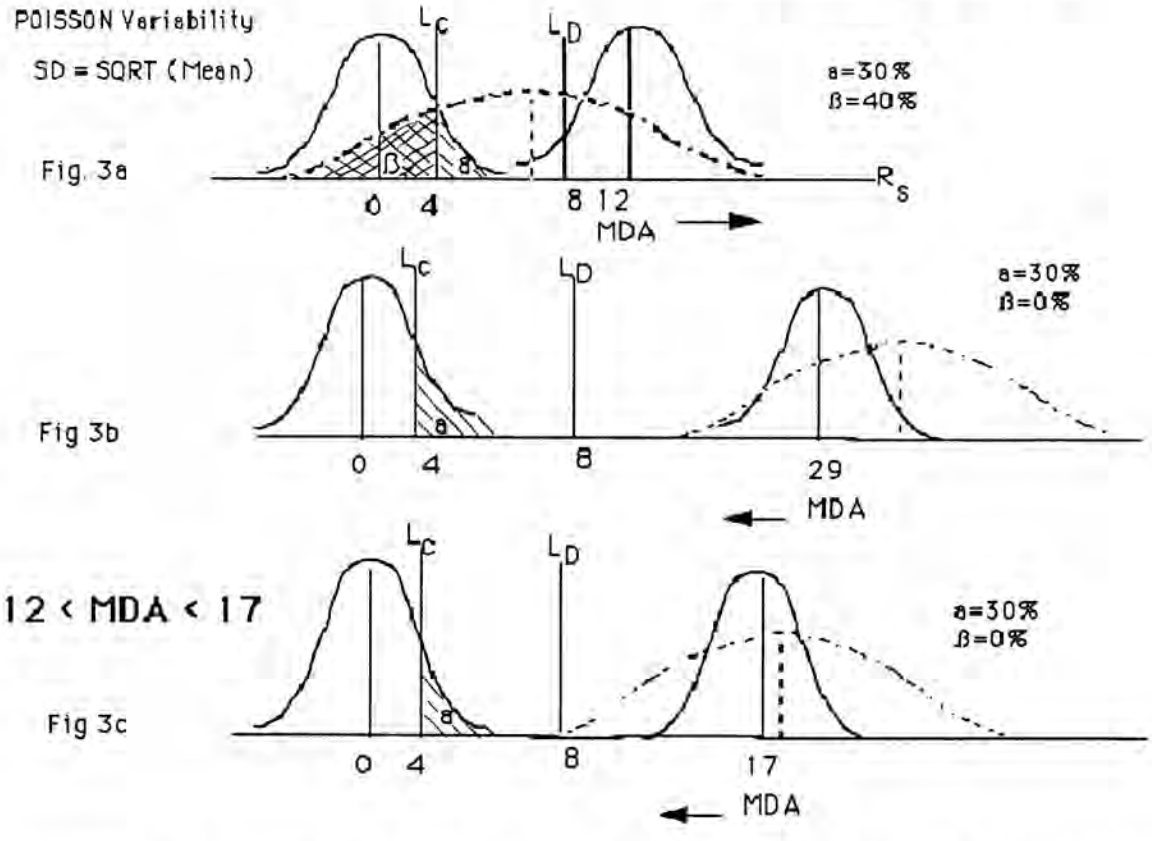

Fig. 3. Comparison of Background, Theoretical, and Actual Source Distributions Using Sample Variability in MDA Equations.

TABLE III
Comparison of MDA Calculations-
Increasing Source Strength

Source Strength (nCi,CPS)	Count Time (sec)	Lc (CPS)	Rs (CPS)	Decision	MDA (CPS)
.80, 11	10	10	22	Rs > Lc	(16,28)
.80, 11	30	10	10	Rs < Lc	l.t. 14)
.80, 11	90	6	9	Rs > Lc	(5,12)
.80, 11	210	4	7	Rs > Lc	(3,12)

TABLE IV
Comparison of MDA Calculations
Increasing Count Time

Source Strength (nCi, CPS)	Count Time (SEC)	Lc (CPS)	Rs (CPS)	Decision	C.I. (CPS)
.80, 12	210	4	7	Rs > Lc	(3, 12)
1.2, 17	210	4	22	Rs > Lc	(19, 24)
2.0, 29	210	4	38	Rs > Lc	(36, 41)
3.2, 46	210	4	49	Rs > Lc	(44, 53)

intervals in fact contained the "true" source value. However, confidence intervals for the smaller sources "barely fit" within the interval.

This result was true for all source activity levels. The intervals narrow (on a percent basis) as the source strength increases, giving a smaller error band for predicting the source with longer count times.

In Table IV, we again see that the theoretical MDA equations predict a correct confidence interval (i.e. contain the "true" source value) except for the ten second count time. Again, we see a smaller L_c with increased count times. Using such a small source (.8 nCi), the confidence intervals are broader (on a percent basis) and do not show consistent narrowing with increased count times as would have been expected with larger sources.

We graphically show the search for the empirical MDA in Figs. 3 and 4. The sensitivity of the MDA theoretical calculations to the Poisson assumption and stable back-

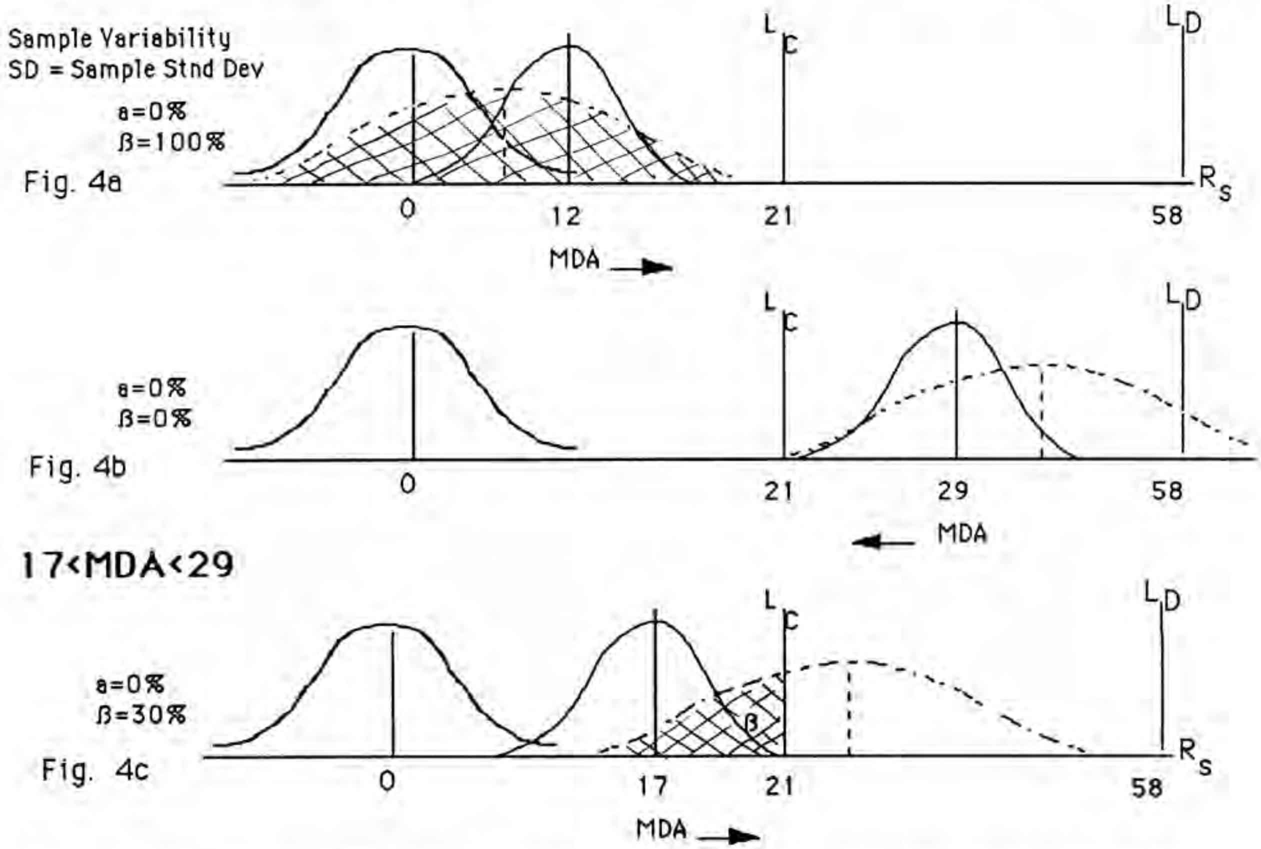

Fig. 4. Comparison of Background, Theoretical, and Actual Source Distribution Using Sample Variability in MDA Equations.

ground assumption is also shown. To read these figures, the two solid distributions (normal curves) represent background (at $R_S = 0$) and the expected known source ($R_S =$ 12 cps, 29 cps, and 17 cps in the respective three graphs labeled a, b, and c). The dashed line is the actual distribution of the calculated net signal ($R_S = R_G - R_B$). As expected, the actual distribution is much broader than the expected distribution and in most cases (R_S = 29 cps in Fig. 3.b and R_S = 17 cps in Fig. 3.c), the average $(\overline{R_S})$ $[\overline{R_S}]$ = 38 cps in Fig. 3.b and $[\overline{R_S}]$ = 22 cps in Fig. 3.c) is larger than the "true" value.

In Fig. 3, the Poisson variability is used in the MDA calculation. In all three cases (Fig. 3.a,b.c), L_c, L_d, and the proportion of false positives (α = 30%) stay the same regardless of the source being counted.

In Fig. 3.a, the proportion of false negatives (β = 40%) indicates that the MDA must be larger than the known source (12 cps) being tested since there are more false negatives than used in the Lc derivation (β = 10%). Thus the MDA shows an arrow toward larger values.

At this level of L_c, 40% of the actual distribution is below L_c, indicating that 40% of the time there is no activity "detected" above background. We know that the true MDA must be higher than 12 cps.

In Fig. 3.b, we use a 29 cps source. There are no (β = 0%) false negatives, indicating that the MDA must be smaller than 29 cps.

In Fig 3.c, a 17 cps source also gives no (β = 0%) false negatives. Time and cost constraints limited the number of sources we could empirically test. We concluded that the true MDA was between 12 cps and 17 cps.

In Fig. 4, the sample standard deviation is used in the MDA calculation. Again, L_c, L_d and the proportion of false positives stay the same in the three cases (Fig. 4.a,b,c). Using actual standard deviation in the MDA equation resulted in a large L_c and a resulting α = 0%. In fact, if the actual distribution of background was used rather than the "assumed Poisson" as shown, L_c probably would have intersected the actual background distribution with 10% of the probability to the right of L_c (i.e., 10% false positives). In Fig. 4.a, using a 12 cps source, there are 100% false negatives. This is to be expected because L_c is so large. We require only 10% false negatives. These results indicate the true MDA must be greater than 12 cps. In Fig. 4.b, using a 29 cps source, there are 0% false negatives, indicating the true MDA must be smaller than 29 cps. At 17 cps, there are 30% false negatives, indicating a true MDA greater than 17 cps. We can conclude that the true MDA is between 17 cps and 29 cps. Note, this MDA interval is larger than the interval calculated under the Poisson assumption.

CONCLUSION

As the nuclear utility industry and its regulators are interested in lower and lower levels of radioactivity to control release limits, more attention is focused on the ability of detection equipment to "see" such small sources. The validity of equipment detection limits focuses on the minimum detectable activity (MDA) value associated with the measurement process.

It has been shown that the MDA calculations in common use today depend on the assumptions used in the measurement process. If the equipment is correctly calibrated, and systematic error sources are removed or diminished as much as possible, statistical theory can be used to test whether the background level is stable and the variability follows a Poisson distribution. If the background passes this test, the MDA equations can be used without modification to provide accurate minimum detectable levels of activity. If the background fails this test, adjustments need to be made to the MDA calculations.

Increasing the count time for source counting theoretically lowers the MDA. However, if the background does not remain stable, increased count times can also increase the likelihood of a change in background, leading to increased background variability, thus increased MDAs. Testing under actual counting conditions needs to be conducted to correctly assess the effects of increased count time on MDA.

Most MDA formulas correctly predict a confidence interval or less-than value for the MDA. The level of precision of these estimates depends on the length of the count time, the source strength, and the validity of the MDA assumptions.

Empirical testing of the MDA assumptions, especially background variability, and incorporating the assumptions into the MDA equations has been shown to give the most accurate detection limits for radiation monitoring equipment.

REFERENCES

1. L.A. CURRIE, "Limits for Qualitative Detection and Quantificative Determination," Anal. Chem., 40:3, 586 (1968).
2. B. ALTSHULER and B. PASTERNACK, "Statistical Measures of the Lower Limits of Detection of a Radioactivity Counter," Health Physics, 9, 293 (1963).
3. L.A. CURRIE, "Lower Limit of Detection: Definition and Elaboration of a Proposed Position for Radiological Effluent and Environmental Measurements,"

NUREG/CR-4007, prepared for the U.S. Nuclear Regulatory Commission (1984).

4. J.C. LOCHAMY, "The Minimum-Detectable-Activity Concept," National Bureau of Standards SP456 (1976).
5. R. LOEVINGER and M. BERMAN, "Efficiency Criteria in Radioactive Counting," Nucleonics, 9:1, 26 (1951).
6. J.J. DONN and R.L. WOLKE, "The Statistical Interpretation of Counting Data from Measurements of Low-level Radioactivity," Health Physics, 32, 1 (1976).