MULTI-OBJECTIVE OPTIMIZATION FOR ROUTING AND SCHEDULING OF RADIOACTIVE MATERIAL SHIPMENTS

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ABSTRACT

Under normal, non-accident conditions low levels of radiation emission from shipments of radioactive materials may occur. The health risk to an individual from these low doses is generally agreed to be small though finite. Nevertheless, many individuals adjacent to a route and traveling on that route may be exposed to low doses of radiation which results in a much larger cumulative dose burden to the population at large than to any individual. Current federal regulations do not require under all circumstances that such shipments be routed and scheduled to minimize the radiation dose to the population at large. A more conservative regulatory philosophy might include consideration of routing and scheduling such shipments so that the general population dose would be as low as reasonably achievable. This work describes a technique for determination of the optimum route and schedule for shipments of radioactive material based on route-specific input data. The analysis technique is structured to allow efficient, simultaneous consideration of multiple objectives with variable weighting factors. Models of transportation systems are optimized via an enhanced network analysis procedure -- reaching. Reaching permits monitoring of the time of arrival of shipments in urban areas to allow minimization of the effects of rush period encounters on accumulated radiation dose, time of travel, and other parameters.

SUMMARY

Current federal regulations for the highway transportation of large quantity shipments of radioactive material do not require under all circumstances that such shipments be routed and scheduled to minimize the radiation exposure dose to the population at large. In fact, one provision of the U. S. Department of Transportation regulations specifies selection under certain conditions of a route "... to reduce time in transit"(1). Intuition might suggest that the longer a radioactive material shipment is in transit, the greater the dose burden to the population at large. However, that rationale is invalid if the population densities and traffic densities along all routes are not the same. Population density and traffic density obviously are not uniform across the nation.

Even under normal, non-accident conditions, low levels of radiation emission from shipments of radioactive material are permitted and do occur, especially in shipments of extremely radioactive materials such as spent fuel. The health risk to an individual from these low doses is generally agreed to be small though finite. Nevertheless, many individuals adjacent to and traveling on a route may be exposed to low radiation doses resulting in a much larger cumulative dose burden to the population at large than to any individual. A more conservative regulatory philosophy might also consider routing and scheduling such shipments so that the general population dose would be as low as reasonably achievable (ALARA). Although not yet generally applied to transportation analysis, the ALARA principle is widely accepted in the field of radiation protection and has been applied to many of the other cases in which people can be exposed to radiation.

One of the primary purposes of the work described here is to develop an analysis technique that allows consideration of multiple objectives (time of travel and radiation dose were selected for the initial models developed here) with the relative importance of the objectives determined by variable weighting factors. The analysis technique is structured to accommodate the addition of other routing and scheduling criteria, such as route specific accident data and shipment security data. A further specific goal of this work is the incorporation of analysis features for evaluation of time of day variation of urban traffic parameters (such as speed and density) and for consideration of those variations in the determination of optimum shipment routes and schedules.

The methodology developed for routing and scheduling analysis consists of three distinct steps. The initial step is the determination of the impedance (weighted sum of time of travel and radiation dose for this preliminary analysis) for each highway segment in the network of interest. These values are then used in the second step -- a Dijkstra shortest path analysis of a straightforward network model representing relative locations of highway intersections. The shortest path analysis yields the lowest accumulated impedance path from each highway intersection to the selected destination ignoring possible rush period encounters. The shortest path analysis is also used to select and evaluate an initial feasible path for the third step of the analysis.

Finally, the results of the shortest path analysis are used in the third step for the more complex analysis of a time/place network that represents both the relative location of highway intersections and the time of arrival at intersections. The time/place network is analyzed using a modified enumeration technique called reaching. The reaching algorithm is enhanced with an elimination by bounds procedure (using the shortest path results) and an elimination by backtracking procedure. The elimination procedures effectively limit the number of possible paths by eliminating those that cannot possibly be better than a current best feasible path and by eliminating all paths that generate loops or circuits. The reaching analysis identifies the route(s) and departure time(s) that will yield the lowest accumulated impedance. These elimination procedures introduce necessary efficiency into the enumeration techniques which would otherwise be impractical for application to even simple actual systems.

PLACE NETWORK MODEL

The first phase of this research was the development of models that appropriately represent the problems to be analyzed. Network models consisting of nodes (intersections) and arcs (highway segments) with constant arc parameters are common. In fact, the initial step in the analysis technique developed here consists of the analysis of such a network temporarily disregarding the added complications of rush period considerations. This preliminary analysis yields the path that minimizes an objective function consisting of a weighted sum of time of travel and radiation dose.

The challenging modeling problem and a primary goal of this work was devising for the next step of the analysis a more sophisticated model to represent the time/place relationships within the network as a function of departure time, time of travel, and rush hour encounters. Details of the time/place network devised for this problem appear below.

The place network model SIMPNET was developed to represent a physical network of highways and intersections for which the highway segment data, such as time of travel and traffic density, were not dependent on the time of day. The term "place" network is used to distinguish this model from a "time/place" model that will be introduced later. The small network, SIMPNET, shown in Fig. 1 illustrates the features of place networks.

External flows are enclosed in brackets and arc parameters are enclosed in parentheses. The set of nodes for a place network with n nodes is represented by N. The set of arcs for a place network with m arcs is represented by M. Individual arcs may be identified as k or (i,j) where i is the origin node of the arc and j is the terminal node of the arc. External flow at node i is represented by b_i. The nodes from which a shipment begins its journey (the source node) will have an external flow of $b_s = +1$ and the node t at which the journey ends (the sink node) will have an external flow of $b_t = -1$. Flow from node i to node j is represented by *f(i,j)* or f_k. These flows are restricted to the values 0 or 1 because the analyses of these place networks are performed for a unit shipment from source to sink. Negative arc flows are not allowed. Furthermore, conservation of flow must be obeyed at each node. No circuits or loops are allowed and arc gains do not occur.

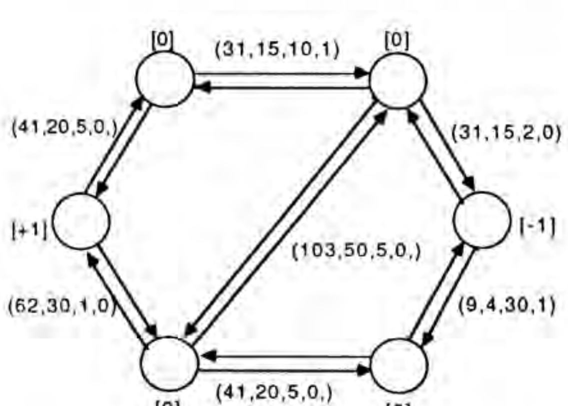

Fig. 1. SIMPNET--Sample Place Network.

The values in parentheses beside each arc in Fig. 1 are parameters that describe that arc. For the network shown, the values represent the time of travel t_k (minutes), arc length l_k (miles), population density p_k (people per square mile),and an urban flag u_k (described later). Time was discretized in the relatively short unit of minutes to minimize round off errors and to yield more precise schedules. For convenience in analyzing the place networks, the network as shown in Fig. 1 is modified to form an expanded network by replacing each single undirected arc with a pair of equal value, opposite direction arcs that are otherwise identical. The arc parameters are positive regardless of the direction of the arc.

The general set of origin nodes o_i for a place network is represented by O. The general set of terminal nodes t_i for a place network is represented by T. The general list of arcs with origin node i is represented by

$$M_{Oi} = [k \mid o_k = i] \qquad \text{(Eq. 1)}$$

The general list of arcs with terminal node i is represented by

$$M_{Ti} = [k \mid T_k = i] \qquad \text{(Eq. 2)}$$

A final arc parameter must be described before the general place network problem can be translated into algebraic form. The goal of the preliminary shortest path analysis is to find the route that yields the minimum value for the objective function. For example, the shortest distance path between nodes 1 and 6 for the SIMPNET network follows the sequence of nodes 1, 2, 5, 6 with arc lengths of 20, 15, and 15 miles for a total distance of fifty miles. The shortest time of travel follows the same path and requires 103 minutes to traverse. The arc parameter of interest for these analyses is, however, a combination of time of travel, t_k, and radiation dose, d_k. This impedance parameter h_k for arc k is defined as

$$h_k = d_k + \lambda t_k \qquad \text{(Eq. 3]}$$

here lambda is a weighting factor used to vary the relative importance of time of travel and radiation dose.

ALGEBRAIC REPRESENTATION OF PLACE NETWORK SHORTEST PATH PROBLEM

The goal of the shortest path analysis is the identification of the path that yields the smallest accumulated impedance. It has also been noted that the flow in each arc of the network must be equal to zero or one and that conservation of flow applies. These goals and constraints may be described algebraically as follows:

$$\text{Min}: \quad \sum_{m}^{k=1} h_k f_k \qquad \text{(Eq. 4)}$$

$$\text{Subj to:} \quad \sum_{k \in M_{Oi}} f_k - \sum_{k \in M_{Ti}} f_k = b_i \qquad \text{(Eq. 5)}$$

$$\text{where} \quad i = 1,2,\ldots,n$$

$$b_s = +1, \; b_t = -1$$

$$b_i = 0 \text{ for } i \neq s, t$$

$$\text{Flow constraint:} \quad f_k = 0, 1. \qquad \text{(Eq. 6)}$$

The specification that $b_i = 0$ for all nodes except the source node s and the sink node t indicates that there are no slack nodes. Subsequent steps in the routing and scheduling analysis are somewhat more complex.

TIME/PLACE NETWORK MODEL

After finding the shortest path through the place network, the next step in the transportation analysis was the addition of time of day variation of urban parameters and the tracking of the time of arrival at each node to determine whether a rush period and an urban area coincide. For this second problem the place network presentation described above was unsuitable. The necessity of continuously monitoring accumulated time of travel suggested the use of an array of time/place states for temporally repeated flow such as that described by Ford and Fulkerson (2). The indices that identify the states are time on one axis and place node on the other axis of a two-dimensional array. The term "state" is used to describe the elements of the time/place array to distinguish them from nodes in the place network

Transit through the SIMPNET place model described above is illustrated with a time/place format in Fig. 2 for a single departure time equal to zero (temporarily disregarding rush periods). The places designated on the horizontal axis represent the nodes of the associated place network already described, and the vertical axis indicates the time at which a state is reached via an indicated path through the time/place array. The time/place state generated by traversing the arc from place node 1 to place node 2 is labeled "b" in Fig. 2, and will be referred to as state (41,2), indicating arrival at place node 2 at time 41.

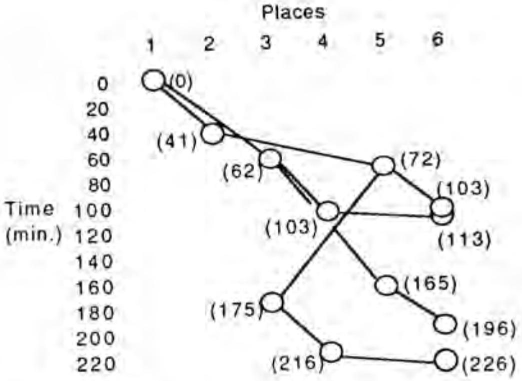

Fig. 2. SIMPNET Time/Place Array for a Single Departure at Time Zero (Rush Periods Disregarded).

The state array generated by the SIMPNET network for three departure times, (0, 20, and 40) would be represented by shifting the state array in Fig. 2 down (indicating a later departure time) and superimposing the result over the original array. Each departure time would have a corresponding source state. Rush periods are again disregarded temporarily to maintain similar time relationships among the states generated. The departure times are represented by t_{dm}. Embedded in the three departure time array would be three "sub-arrays" identical to the single departure time array illustrated in Fig. 2. Each sub-array can include one or more sink states. A sink state is generated when a path through the time/place array reaches the place node designated as the sink. The time of arrival and the sink node number define the sink state.

Rush periods are the final feature of real highway transportation networks to be incorporated in the graphical time/place array representation. For the example SIMPNET network, each complete cycle (analogous to a 24 hour weekday cycle for actual highway networks) is assumed to last 60 minutes. Two rush periods are assumed to occur from minutes 20 to 25 and from minutes 40 to 45. That is, if a shipment traversing SIMPNET arrives at the origin node of an urban arc during one of these rush periods, the travel time and the traffic density on the urban arc will be doubled to account for higher trafic density. The 60-minute cycle is continuously repeated so that the rush periods occur at the same time relative to the beginning of each cycle. The variables r_{t1} and r_{t2} are defined as the rush period indicators. These variables are equal to 1 for the times that comprise their respective rush periods and their values are zero otherwise. Mathematically the rush periods may be described as follows:

$$r_{t1}=1 \text{ for } (20+60i) \leq t \leq (25+60i) \qquad \text{(Eq. 7)}$$

$$r_{t2}=1 \text{ for } (40+60i) \leq t \leq (45+60i)$$

$$\text{where } i = 0,1,2,\ldots$$
$$t = 1,2,3,\ldots$$

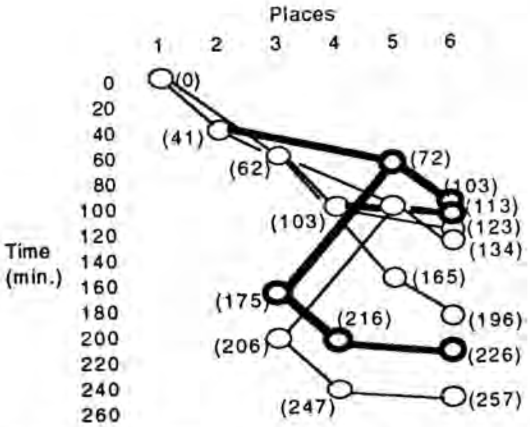

Fig. 3. SIMPET Time/Place Array for a Single Departure at Time Zero (Rush Periods Included).

Whenever the time of arrival at an urban node coincides with one of these rush periods, the urban arc parameters are appropriately altered.

The urban flag u_k shown for each arc in Fig. 1 indicates whether the arc is designated as an urban arc. A value of 1 indicates an urban arc, and a value of 0 indicates a non-urban arc.

Consideration of rush periods in generating the time/place state array can obviously affect the location of the states in the time/place array. Whenever an urban arc is encountered during a rush period, the time required to reach the terminal place node of that arc is assumed to be twice the normal value. Also affected will be the impedance for that arc which is a function of time of travel and radiation dose -- both of which are increased by rush hour encounters. The population doses increase because the slower speed increases the time of exposure and the greater traffic density increases the number of people exposed.

The time/place state array for the SIMPNET network with the urban arcs designated in Fig. 1, with rush periods designated by Equation 7, and with a single departure time of zero is illustrated in Fig. 3. The bold lines and circles highlight the portions of the non-rush period array that are no longer generated when the rush period encounters are included.

ALGEBRAIC REPRESENTATION OF THE TIME/PLACE STATE ARRAY SHORTEST PATH PROBLEM

The fundamental concepts underlying the algebraic representation of the time/place state array are similar to those underlying the algebraic representation of the simpler place network. The primary objective is the minimization of the sum of the products of arc flows and arc impedances. However, the arc impedances vary when rush period encounters occur. To account for this time dependence of arc parameters, the flow in each arc is defined for each time interval t. The flow in arc k at time t is designated f_{kt}. The impedance for non-rush period conditions is still designated h_k, and the amount by which the impedance increases during rush periods is represented by H_k. Of course, the value of H_k for non-urban arcs is zero because the impedance for those arcs remains constant regardless of time. The final parameters necessary to describe the minimization objective are the rush hour indicators r_{t1} and r_{t2} defined by Eqs. 7. The minimization objective can be expressed as shown in Eq. 8. Whenever either r_{t1} or r_{t2} is equal to 1 (only one can be non-zero at any time) and an urban arc has been encountered, the term $H_k f_{kt}(r_{t1} + r_{t2})$ is non-zero. If $H_k = 0$ the arc is not an urban arc. If both r_{t1} and r_{t2} are equal to zero a rush period is not occurring. If $f_{kt} = 0$ no flow is occurring at time t on arc k. The second term in the minimization equation above is non-zero only if none of the three conditions described is present.

The conservation of flow constraints are also complicated somewhat by the time dependence of the flow variable f_{kt}. Conservation of flow must be satisfied at all nodes and at all times. That is, the flow out of node i at time t must be equal to the flow into node i at time t. The time of travel from place node j to place node i is t(ji) time units. Therefore, the flow into node i at time t from node j, departed t(ji) time units prior to arriving at node i. The external flows b_{it} into the source state (or states for multiple departure times) at time t and out of the sink state (or states for multiple departure times) at time t must be balanced by arc flows. The conservation of flow constraints can, therefore, be expressed as shown in Eq. 9. The first term in the conservation of flow equation represents the total flow departing node i on network arcs at time t. The second term represents the total flow entering node i on network arcs at time t. Each entering flow departed from an adjacent node j exactly t(ji) minutes earlier so it would arrive at time t. The symbol b_{it} for i = S represents the external flow into source state (S,t). Likewise, the symbol b_{it} for i = T represents the external flow out of sink state (T,t).

As with the place network and for the same reason, all flows in the time/place array are restricted to the values 0 and 1. The complete algebraic representation of the time/place array problem appears below:

An approach for optimizing the time/place state array described algebraically by the preceding equations is

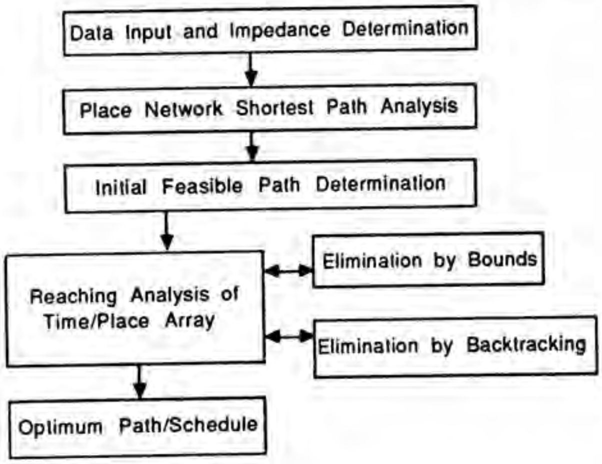

Fig. 4. Generalized Flowchart for Analysis Technique.

presented below.

$$\text{Min: } \sum_{m}^{k=1}[h_k f_{kt} + H_k f_{kt}(r_{t1}+r_{t2})] \qquad \text{(Eq. 8)}$$

$$\text{where} \quad t = 0,1,2,\ldots$$

$$\text{Subj to: } \sum_{k \in M_{Oi}} f_{kt} - \sum_{k \in M_{Ti}} f_{k;t-t(ji)} = b_{it} \qquad \text{(Eq. 9)}$$

$$\begin{aligned} \text{where} \quad & t = 0,1,2,\ldots \\ & i = 1,2,\ldots,n \\ & b_{it} = +1 \text{ for } i = S \text{ and } t + t_{dm} \\ & b_{it} = -1 \text{ for } i = T \end{aligned}$$

$$\text{Flow constraint:} \quad f_{kt} = 0,1. \qquad \text{(Eq. 10)}$$

OVERVIEW OF THE SOLUTION TECHNIQUE

Figure 4 illustrates the major steps of the procedure devised to analyze the network models for spent fuel shipping routes and schedules. The weighted sum of time of travel and general population radiation dose is referred to as the impedance of an arc or path and the term lowest impedance path will be used interchangeably with the term shortest path. The material in this section provides an overview of the primary elements of the analysis technique -- the reaching algorithm, the elimination by bounds procedure, the backtracking procedure, and the Dijkstra shortest path algorithm.

The reaching algorithm is the central element of the transportation analysis developed (3). Beginning with the initial states defined for the problem, the algorithm generates possible states of the time/place array and checks the parameters of the states as they are generated to determine the lowest impedance path to each state from a source state.

As each new state Y(t,n) is generated, a second test is performed. The test determines whether the lowest impedance path from a source state to the state Y(t,n) plus the lower bound of the path from Y(t,n) to a sink state can possibly improve the best available source-to-sink path for the time/place array. This elimination procedure is referred to as elimination by bounds (3). The lower bound Z_{lb} for each state Y(t,n) is the lowest impedance path possible from state Y(t,n) to a sink state. The value of Z_{lb} for each state is determined by relaxing the rush period constraint on the time/place array (i.e.,ignoring rush period effects) and then analyzing the resulting, less complex place network with a Dijkstra shortest path analysis. The impedance of all paths from a state Y(t,n) to sink states can only increase when rush period constraints are added. Therefore, the shortest path ignoring rush periods provides the lower bound for the impedance to reach a sink state. Any path through the time/place network that reaches the sink node (regardless of the time of arrival) generates a sink state.

The other important test performed during the reaching analysis determines whether the place node n associated with a state Y(t,n) about to be generated has been encountered at some earlier state in the path that leads to Y(t,n). If it has, the state under consideration is not generated. This is the backtracking procedure mentioned previously.

The primary comparisons and eliminations described provide powerful mechanisms to limit the number of states generated for time/place state arrays by the reaching algorithm. Without the eliminations, analysis of the time/place array would amount to generation of all possible states followed by identification of the sink states and associated paths with the lowest accumulated impedance. Detailed algorithms to perform the reaching and elimination procedures have been developed and incorporated in a FORTRAN-77 computer code for transportation analysis (4). The code operates on desktop computers with 80286 processors and complementary math coprocessors.

Application of the reaching algorithm (with elimination procedures) to the SIMPNET example with three departure times yielded a total of nine states. Without the elimination procedures at least thirty-three states would have been generated. In a larger sample network with about 100 arcs and 100 nodes, an estimated 60,000 states would have been generated using the reaching technique without the

elimination procedures. Using those procedures reduced the number of states generated to only a few hundred.

CONCLUSION

The primary features of the routing and scheduling analysis technique developed here are (a) use of data specific to individual highway segments for routing and scheduling optimization, (b) use of a time/place network model that allows simultaneous scheduling and routing analysis, (c) use of an enhanced network analysis algorithm -- reaching -- that with elimination techniques efficiently optimizes routing and scheduling including rush period considerations, and (d) consideration simultaneously of multiple decision variables in determining optimum routes and schedules for radioactive material shipments.

Although the technique developed and described is discussed in the context of spent fuel transportation, the methodology and algorithms are adaptable to other issues. Route-specific parameters such as adjacent population density, traffic density, and accident rates can be useful in the routing of virtually any hazardous material. The selection of routes exhibiting low accident rates could reduce the probability of accidents and subsequent release of hazardous cargoes. Using routes with lower adjacent population densities and traffic densities can reduce the consequences of releases that may occur. Similar considerations can be of interest for shipments of explosives and weapons.The scheduling features of the analysis technique have even more extensive potential. The analysis technique can be applied to the shipment of any commodity to determine the route and schedule that will yield the shortest distance route, the shortest time of travel route, or even a combination of shortest distance and shortest time of travel. The most interesting feature of this analysis is the explicit consideration of rush period effects in urban areas. The network to be analyzed could represent any type of highway system, any area, or some other transportation network (for example, railroads). In fact, any time-repeated network with deterministic, time-varying parameters can be analyzed using this technique. Examination of the literature on transportation and network analysis yielded no indication that a technique for treating such networks has been previously developed.

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