ACCELERATED BETA DECAY FOR NUCLEAR WASTE DISPOSAL

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ABSTRACT

The concept of the acceleration of the rates of forbidden nuclear beta decays is introduced. Virtually the entire burden of waste disposal arising from fission products is associated with forbidden beta decays. The physical mechanism by which an applied electromagnetic field can remove forbiddenness from beta decay does not require energetic field photons, and so it is proposed to use low frequency fields to accelerate forbidden decays. A brief review of the quantum-mechanical formalism for the calculation of electromagnetically enhanced beta decay is given. With stress on strontium-90 as an example, it is concluded that substantial lifetime reduction can be achieved. The use of low-frequency fields makes possible decay enhancement at relatively low cost. It is proposed that useful power be generated by electromagnetic beta decay acceleration to recover the costs of thereby disposing of high-level fission product wastes.

INTRODUCTION

Nuclear beta decay is termed "forbidden" when a strong inhibition of the decay process occurs as a result of the violation of the "selection rules" of beta decay. These selection rules are such that a decay is allowed only when initial and final nuclear quantum states differ by either zero or one unit of angular momentum (measured in units of $\hbar$, Planck's constant divided by 2π); and when the intrinsic parities of the initial and final states are the same. There are orders of forbiddenness depending on the degree to which the selection rules are violated. The approximately thirty-year halflives of ^{90}Sr and ^{137}Cs would be of the order of one day were it not that both these materials have first-forbidden beta decays. Examples of second-forbidden beta decays are ^{135}Cs (2×10^6 years) and ^{99}Tc (2×10^5 years).

The basic mechanism by which an electromagnetic field can influence forbidden beta decay comes from the property that each photon of a plane-wave field carries one unit of angular momentum (J), and causes a change in the intrinsity parity (P) of a quantum state with which it interacts. This is often expressed by saying that $J^P=1^-$ for a photon. This has the consequence that the intercession of a single photon removes a single degree of forbiddenness from beta decay.

A vital point to observe is that a photon has the property $J^P=1^-$ irrespective of its energy, i.e., without regard to the frequency of the field. This suggests the use of low-frequency fields, with concomitant small input energy $\hbar\omega$ from each photon. All of the necessary decay energy is initially present in the parent nucleus. There is no need to supply energy to make the beta decay occur. Only angular momentum and parity are necessary. Because no energy need be supplied by the field, a substantial amount of beta decay energy can be emitted at the cost of a very small amount of energy in the inducing field. This fact has obvious practical implications.

THEORY

Although low-energy photons can supply angular momentum and parity just as well as high-energy photons, there will be a problem with loss of coupling strength between the field and the beta decay system because a low frequency field is so far off resonance with levels in the nuclear system. The solution to this problem of strength of interaction is to employ intense fields. The parameter which measures the interaction between a charged particle and a plane-wave monochromatic electromagnetic field is the intensity parameter

$$z_f = \frac{e^2a^2}{2m^2c^2}, \tag{1}$$

where e is the charge of the particle of mass m, a is the amplitude of the Coulomb-gauge vector potential $\vec{A}$ of the field, and c is the velocity of light. Alternate, and perhaps even more instructive ways to express z_f are

$$z_f = \frac{e^2|\vec{E}|^2}{2m^2c^2\omega^2} = \frac{e^2|\vec{B}|^2}{2m^2\omega^2}, \tag{2}$$

where $\vec{E}$ and $\vec{B}$ are the electric field and magnetic induction vectors of the plane-wave field of circular frequency ω. In the form given in Eq. (2), the advantage of low frequency for achieving a large intensity parameter is evident. To attain specific intense-field effects (i.e., to achieve an intensity at which perturbation theory fails), z_f must have a value of about 10^{-1} or greater. This will be discussed further in the section on practical applications.

The physical importance of the intensity parameter can be shown in a very striking way. The quantum-mechanical interaction energy of an electron with a plane-wave field is $-e\vec{A}\cdot\vec{p}/m$, when expressed in Coulomb gauge, where $\vec{p}$ is the total momentum operator. The magnitude of this interaction term as compared to the rest energy of the electron is

$$\frac{1}{mc^2}\left|\frac{e\vec{A}\cdot\vec{p}}{m}\right| \approx \left|\frac{e\vec{A}}{mc}\right|\left|\frac{\vec{p}}{mc}\right| \approx \frac{e|\vec{A}|}{mc} = (2z_f)^{1/2}. \quad (3)$$

In Eq. (3), the result is attained using Eq. (1) and the fact that the beta particle momentum is typically of order mc in beta decay. Equation (3) demonstrates that the properties of an electron will be radically altered by its interaction with the field when z_f is of the order of unity. Furthermore, when the interaction term is of the same order as the rest energy, it is clear that perturbation theory is not valid.

As an example of the profound consequences arising from intense field effects, consider the customary relativistic energy-momentum relationship for an electron

$$p^\mu p_\mu = m^2c^2 , \quad (4)$$

where p^μ is the four-momentum vector with time component E/c and space component $\vec{p}$. In the presence of a plane-wave field this is altered to[1]

$$(p^\mu - nk^\mu)(p_\mu - nk_\mu) = m^2c^2(1+z_f) , \quad (5)$$

where k_μ is the four-momentum propogation vector of the electromagnetic field with time component $\hbar\omega/c$ and space component $\hbar\vec{k}$, and n is any integer. Equation (5) can be viewed as meaning that an electron in interaction with the field behaves kinematically as if it carried any number of photons with it in intimate combination.

The simplest way to formulate the theory of electromagnetically enhanced beta decay is to choose the "EF" (electric-field) gauge for expression of the electromagnetic field. When the EF gauge is used, the wave function for the emitted beta particle in interaction with the field contains all the essential physics of electromagnetically enhanced beta decay. Field interaction directly with the nucleus can be neglected.[2] Results calculated in this way for induced transition probabilities agree exactly with those calculated in the Coulomb gauge,[3] although the Coulomb gauge calculation requires that interaction of the field with both the beta particle and the nucleus be included.

The transition probability for electromagnetically enhanced beta decay divides naturally into three additive portions corresponding to: a direct coupling of the field to the decay electron; a coupling to the spin of the electron; and an interference between the two mechanisms. Generally, the first of these three contributions is the most important, but all are of the same order of magnitude. Only the direct term is given here, since it is sufficient for the understanding of the results of the theory. This transition probability per unit time is

$$W_1 = \frac{G^2m^5c^4}{(2\pi)^5\hbar^7}\int d\varepsilon\, \varepsilon(\varepsilon^2-1)^{1/2}\int d\Omega \sum_n\left[\varepsilon_o - \varepsilon - (\eta - n)\frac{\hbar\omega}{mc^2}\right]^2$$

$$\times \frac{1}{nd}\left\{\frac{1}{[1-(d-b)^2]^{1/2}}\left[\left|\exp\left[-i(d-b)\frac{e\vec{a}\cdot\vec{r}}{\hbar} - i(\eta-n)\vec{k}\cdot\vec{r}\right]\right|^2_{fi}\right.\right.$$

$$\left. + \kappa^2\left|\exp\left[-i(d-b)\frac{e\vec{a}\cdot\vec{r}}{\hbar} - i(\eta-n)\vec{k}\cdot\vec{r}\right]\vec{\sigma}\right|^2_{fi}\right]$$

$$+ \frac{1}{[1-(d+b)^2]^{1/2}}\left[\left|\exp\left[i(d+b)\frac{e\vec{a}\cdot\vec{r}}{\hbar} - i(\eta-n)\vec{k}\cdot\vec{r}\right]\right|^2_{fi}\right.$$

$$\left.\left. + \kappa^2\left|\exp\left[i(d+b)\frac{e\vec{a}\cdot\vec{r}}{\hbar} - i(\eta-n)\vec{k}\cdot\vec{r}\right]\vec{\sigma}\right|^2_{fi}\right]\right\} . \quad (6)$$

In Eq. (6), G is the Fermi coupling constant of beta decay; ε is a dimensionless beta particle energy (expressed in units of mc^2); Ω is the solid angle into which the electron is emitted; ε_o is the dimensionless nuclear energy difference between initial and final states,

$$\varepsilon_o = (E_i - E_f)/mc^2 ; \quad (7)$$

η is a dimensionless intense-field momentum coefficient given by

$$\eta = \frac{z_f m^2c^2}{2p^\mu k_\mu} ; \quad (8)$$

b is another such coefficient defined by

$$b = \frac{e\vec{p}\cdot\vec{a}}{2z_f m^2c^2} , \quad (9)$$

where $\vec{a}$ is the vector amplitude of the Coulomb gauge vector potential; d is defined by

$$d = \left[b^2 + \frac{1}{2}(1-n/\eta)\right]^{1/2} ; \quad (10)$$

κ is the ratio of the strength of the axial vector coupling to the vector coupling in beta decay, which has the experimental value 1.23±0.01[4]; and the components of $\vec{\sigma}$ are the standard Pauli spin matrices. The squared absolute values with subscripts fi are squared transition matrix elements. Despite the complexity of Eq. (6), its significance can be appreciated if it is compared with the usual expression for nuclear beta decay in which lowest order lepton momentum terms are retained,

$$W = \frac{G^2 m^5 c^4}{2\pi^3 \hbar^7} \int d\varepsilon\ \varepsilon\ (\varepsilon^2-1)^{\frac{1}{2}} (\varepsilon_0-\varepsilon)^2 \left[|\exp[i(\vec{p}+\vec{p}_\nu)\cdot\vec{r}/\hbar]|^2_{fi} + \kappa^2 |\exp[i(\vec{p}+\vec{p}_\nu)\cdot\vec{r}/\hbar]\vec{\sigma}|^2_{fi} \right] \quad . \qquad (11)$$

The only new quantity in Eq. (11) is the antineutrino momentum $\vec{p}_\nu$.

An essential feature in the comparison between Eq. (6) and Eq. (11) is the quantity contained in the transition matrix elements. In Eq. (11), the quantity can be expanded as

$$\exp[i(\vec{p}+\vec{p}_\nu)\cdot\vec{r}/\hbar] \approx 1 + i(\vec{p}+\vec{p}_\nu)\cdot\vec{r}/\hbar \ . \qquad (12)$$

The first term in Eq. (12) simply leads to the result for allowed beta decay if the initial and final nuclear states are such that the selection rules are satisfied. If the beta decay is first-forbidden, the first term in Eq. (12) makes no contribution, and the transition probability arises from the second term. The presence of $\vec{r}$ in the second term in Eq. (12) causes a change in parity, and the scalar product introduces a first-order spherical harmonic corresponding to a unit of orbital angular momentum. Thus, selection rules from the second term are appropriate for a first-forbidden transition. The relative order of magnitude of the second term in Eq. (12) is given by

$$O(|(\vec{p}+\vec{p}_\nu)\cdot\vec{r}/\hbar|) = R_0\ mc/\hbar = R_0/\lambda\!\!\!^-_c \ , \qquad (13)$$

where R_0 is the nuclear radius, and $\lambda\!\!\!^-_c$ is the electron Compton wavelength. The transition probability per unit time goes as the square of this quantity, and since

$$(R_0/\lambda\!\!\!^-_c)^2 \approx 10^{-4} \ ,$$

one has the known result that first-forbidden beta decay occur about 10^4 times more slowly than allowed decays.

In like fashion to the above analysis, a typical transition matrix element in Eq. (6) has the quantity

$$\exp[-i(d-b)e\vec{a}\cdot\vec{r}/\hbar - i(\eta-n)\vec{k}\cdot\vec{r}] \approx 1 - i(d-b)e\vec{a}\cdot\vec{r}/\hbar - i(\eta-n)\vec{k}\cdot\vec{r} \ . \qquad (14)$$

As with Eq. (12), the first term in Eq. (14) makes no contribution if the transition is forbidden, but the other two terms will lead to significant results for first-forbidden transitions. (It should be noted that exactly the same lepton terms shown in Eq. (12) should also appear in Eq. (6) and Eq. (14). They have been omitted for simplicity.) The last two terms in Eq. (14) are of relative magnitude $R_0/\lambda\!\!\!^-_c$ for intense electromagnetic fields. This is just like the lepton momentum terms in Eq. (12). Therefore the field can supply angular momentum and parity in a fashion and to a degree equivalent to the usual forbidden beta decay terms. The second term on the right-hand side of Eq. (14) is what is called a transverse field contribution, and the third term is a longitudinal field contribution.

RESULTS

It has just been shown that induced electromagnetic beta decay will be essentially of the same order of magnitude as conventional forbidden beta decay, just based on a simple comparison of transition matrix elements. The matrix element comparisons are not exact, however, and there are field-dependent factors in Eq. (6) apart from those contained in the matrix elements. It is necessary to carry out an explicit computation with Eq. (6) to find the actual consequences of electromagnetically accelerated beta decay. An important qualitative feature of intense-field processes is that a field intensity exists corresponding to a maximum transition probability. Further increase in intensity causes a decline in transition probability. The precise value of this maximum intensity depends on the particular process, but it occurs generally in the vicinity of $z_f=1$.

For the example of ^{90}Sr, it is found that the electromagnetically induced transition probability per unit time calculated at optimum field intensity gives a result equivalent to a halflife somewhat under ten years. When combined with the natural halflife of 28.6 years, and considering calculational uncertainties in the induced halflife, the overall result is that ^{90}Sr decay can be induced to occur with a halflife of five to ten years.

An immediate practical question which arises concerns the means of attaining a z_f value of unity. From Eq. (2), a low-frequency source is favored. The requisite value of z_f is achievable with lasers, but only in a small volume, and only with a pulsed source. With rf (radio-frequency) fields, it is possible to get the required intensity in a large volume, on a continuous basis, and with a very reasonable input power requirement.

APPLICATIONS

The most obvious application of the process of electromagnetically accelerated nuclear beta decay is for the disposal of nuclear fission products. The ^{90}Sr example given above is the only numerical result calculated to date, but it indicates an acceleration by a factor of four or so. In general, it appears that acceleration by as much as one order of magnitude may be possible. This suggests a power density in the waste high enough to make power production an attractive possibility as a concomitant to waste disposal.

Suppose a power plant fueled with ^{90}Sr is constructed in which the accelerated halflife is ten years. The density of fuel nuclei in a solid fuel will be about 10^{28} nuclei/m^3. The recoverable energy from each ^{90}Sr decay is about 1.4 MeV, or 2.2×10^{-13} J. This amount is arrived at by assuming that half of the total beta decay energy can be recovered in the 0.546 MeV decay of ^{90}Sr to ^{90}Y, followed by the 2.279 MeV decay from ^{90}Y to ^{90}Zr. The remaining energy is lost to the antineutrinos, which deposit essentially none of their energy in the system. The above figures lead to a power density of 5×10^6 W/m^3.

This power density result may be compared with a typical fission plant power density of 10^8 W/m^3 in the core, or 5×10^4 W/m^3 when averaged over the containment vessel. Estimates for projected large, magnetic-confinement fusion power plants give power densities of 10^6 W/m^3. In either case, the enhanced fission-product beta decay power density appears to be favorable.

Now consider the ^{90}Sr fuel to be emplaced in the form of a dielectric solid between the conductors of a coaxial transmission line. For a standard 50-ohm line (ratio of outer to inner conductor diameter of 2.3), the thermal power produced in the fuel would be

$$P_{th} = (1.3x10^7)\ r_o^{\ 2}\ \ell, \qquad (15)$$

where r_o is the radius of the outer conductor, and ℓ is the length of the transmission line. The ratio of this produced power to the rf power traveling through the line is

$$\frac{P_{th}}{P_{in}} = \frac{(5.4x10^{12})\ell}{\varepsilon_r^{3/2}\ \nu^2}\ , \qquad (16)$$

where ε_r is the relative dielectric constant, and ν is the frequency of the rf power. If one considers a thermal efficiency of 40%, and presumes that both P_{th} and P_{in} are collected and converted to electric power, then one may calculate reactor sizes for a specific ratio of electric power produced to input power. For example, if $\varepsilon_r = 4$, $\nu = 2x10^6$ Hz is assumed, and an output-to-input power ratio $P_{e\ell}/P_{in} = 10$ is demanded, then

$$\frac{P_{e\ell}}{P_{in}} = 0.4\left(\frac{P_{th}}{P_{in}} + 1\right) = 10\ , \qquad (17)$$

and Eqs. (16) and (17) indicate that a length of 140 m is required. The properties of low-frequency coaxial transmission lines are such that this transmission line length need not refer to a straight section. The basic TEM field mode inside the line is maintained even if the line is coiled.

Equation (16) shows that the length is the only size parameter which determines power ratios. The total power produced from a ^{90}Sr enhanced-decay reactor follows from

$$P_{e\ell} = 0.4\ P_{th}(1 + P_{in}/P_{th})\ . \qquad (18)$$

Equations (15) and (18) show that a commercial six-inch 50-ohm line would produce 4 MW(eℓ). If, for example, the outer coax radius was one meter, then $P_{e\ell}$ = 700 MW for ℓ = 140 m. In either case, maximum electric field strength in the coax is well within commercial limits, and electrical breakdown would not be a problem.

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